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(*Frontispiece*)

A PRACTICAL TREATISE
ON
SUSPENSION BRIDGES

THEIR DESIGN, CONSTRUCTION AND ERECTION

BY

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WITH APPENDIX:

DESIGN CHARTS FOR SUSPENSION BRIDGES

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PREFACE

This book has been planned to supply the needs of practicing engineers who may have problems in estimating, designing or constructing suspension bridges, and of students who wish to prepare themselves for work in this field. The aim has been to produce an up-to-date, practical handbook on the subject, distinguished by simplicity of treatment and convenience of application.

In the first division, on Stresses in Suspension Bridges, the formulas have been corrected to conform to modern practice, and reduced to their simplest and most convenient form for direct application by the designing engineer. The formulas are supplemented by curves for their expeditious solution, and by alternative graphical methods for determining stresses.

The second division, on Types and Details of Construction, presents data and illustrations to assist the designing engineer in the selection of type of suspension bridge and in the determination of proportions, specifications, and details for the various elements of suspension construction.

The third division, on Typical Design Computations, gives numerical examples of suspension designs of different types worked out by methods that have proved most efficient in the author's practice. The designing engineer will find here the formulas to be used in each successive step of the design, and the practical methods of applying them, with tabulations, graphs and short-cuts.

The fourth division, on Erection of Suspension Bridges, describes and illustrates the successive stages in the erection of representative structures, from towers to trusses. The operations of stringing wire cables are presented in detail, with an outline of the computations for adjustment and control.

Methods of erecting eyebar chains and other types are also described and illustrated.

The Appendix presents a series of design charts, specially devised for this book, for the expeditious proportioning of suspension bridges. These charts give quickly and accurately the governing stresses throughout any span, saving the time and labor of applying the stress formulas otherwise required.

The author desires to express his indebtedness to his associate, Mr. Holton D. Robinson, for reviewing the manuscript on Erection; and to the Department of Plant and Structures of New York City for many courtesies extended.

D. B. Steinman.

New York City
August 1, 1922.

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A PRACTICAL TREATISE

ON

SUSPENSION BRIDGES

THEIR DESIGN, CONSTRUCTION AND ERECTION

CHAPTER I

STRESSES IN SUSPENSION BRIDGES

SECTION I.—THE CABLE

1. Form of the Cable for Any Loading.—If vertical loads are applied on a cable suspended between two points, it will assume a definite polygonal form determined by the relations between the loads (Fig. 1*a*).

The end reactions (T_1 and T_2) will be inclined and will have horizontal components H. Simple considerations of static equilibrium show that H will be the same for both end reactions, and will also equal the horizontal component of the tension in the cable at any point. H is called the horizontal tension of the cable

Let M' denote the bending moment produced at any point of the span by the *vertical* loads and reactions, calculated as for a simple beam. Since H, the horizontal component of the end reaction, acts with a lever-arm y, the total moment at any point of the cable will be

$$M = M' - H \cdot y. \qquad (1)$$

This moment must be equal to zero if the cable is assumed to be flexible. Hence,

$$M' = H \cdot y, \qquad (2)$$

and

$$y = \frac{M'}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) gives the ordinates to the cable curve for any loading, if the horizontal tension H is known. Since H is constant, the curve is simply *the bending moment diagram for the applied loads*, drawn to the proper scale. The scale for con-

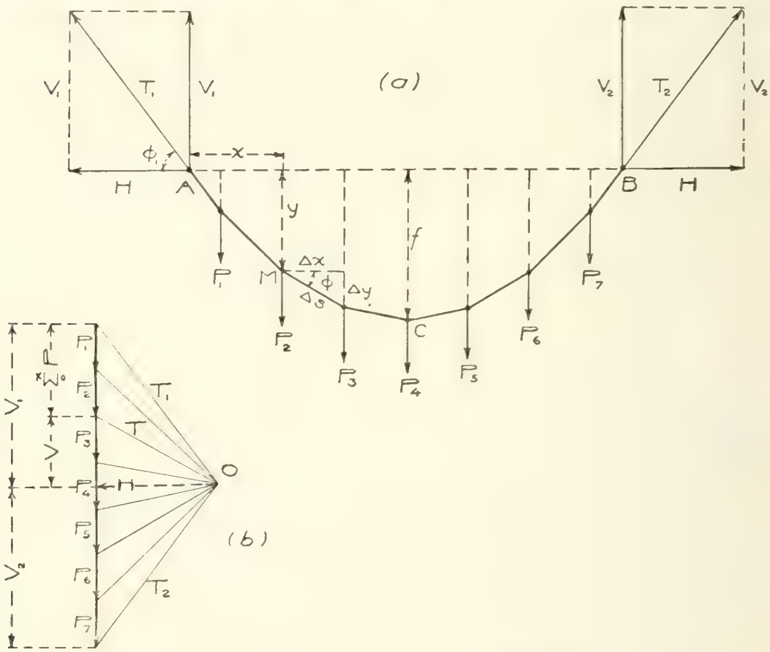

FIG. 1.—The Cable as a Funicular Polygon.

structing this diagram is determined if the ordinate of any point of the curve, such as the lowest point, is given. If f is the sag of the cable, or ordinate to the lowest point C, and if M_c is the simple-beam bending moment at the same point, then H is determined from Eq. (2) by

$$H = \frac{M_c}{f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To obtain the cable curve graphically, simply draw the equilibrium polygon for the applied loads, as indicated in Fig. 1

(a, b). The pole distance H must be found by trial or computation so as to make the polygon pass through the three specified points, A, B, and C. The tension T at any point of the cable is given by the length of the corresponding ray of the pole diagram. H, the horizontal component of all cable tensions, is constant. By similar triangles, the figure yields

$$T = H \cdot \frac{\Delta s}{\Delta x} = H \cdot \sec \phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where ϕ is the inclination of the cable to the horizontal at any point. It should be noted that the tensions T in the successive members of the polygon increase toward the points of support and attain their maximum values in the first and last members of the system.

If V_1 is the vertical component of the left end reaction, the vertical shear at any section x of the span will be

$$V = V_1 - \sum_0^x P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This will also be the vertical component of the cable tension at the same point. By similar triangles,

$$V = H \cdot \frac{\Delta y}{\Delta x} = H \cdot \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

(This relation is also obtained by differentiating both members of Eq. 2). Combining Eqs. (6) and (7), we may write

$$\frac{\Delta y}{\Delta x} = \frac{V_1 - \sum_0^x P}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the loads are continuously distributed, the funicular polygon becomes a continuous curve. If w is the load per horizontal linear unit at any point having the abscissa x, Eq. (8) becomes

$$\frac{dy}{dx} = \frac{V_1 - \int_0^x w \cdot dx}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

from which (by differentiation) we obtain the following as the differential equation of the equilibrium curve:

$$\frac{d^2y}{dx^2} = -\frac{w}{H}. \qquad (10)$$

For any given law of variation of the continuous load w, the integration of Eq. (10) will give the equation of the curve assumed by the cable.

2. The Parabolic Cable.—For a uniform distributed load, the bending moment diagram is a parabola. Consequently, by Eq. (3), if a cable carries a uniform load (w per horizontal linear unit), the resulting equilibrium curve will be a parabola.

The maximum bending moment in a simple beam would be

$$M_c = \frac{wl^2}{8}.$$

Substituting this value in Eq. (4), the horizontal tension is determined:

$$H = \frac{wl^2}{8f}. \qquad (11)$$

To obtain the equation of the curve, integrate Eq. (10). With the origin of coordinates at the crown, the integration yields

$$y = \frac{wx^2}{2H}. \qquad (12)$$

Substituting the value of H from Eq. (11), we obtain the equation of the parabola,

$$y = 4f\frac{x^2}{l^2}. \qquad (13)$$

If the origin of coordinates is taken at one of the supports (as A, Fig. 1), the equation becomes,

$$y = \frac{4fx}{l^2}(l-x). \qquad (14)$$

The maximum tension in the cable, occurring at either support, will be

$$T_1 = \sqrt{H^2 + (\tfrac{1}{2}wl)^2},$$

or, by Eq. (11),

$$T_1 = \frac{wl^2}{8f}\sqrt{1+16n^2}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

where n denotes the ratio of the sag f to the span l:

$$n = \frac{f}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

Equation (15) may also be derived from Eq. (5) by noting that the inclination of a parabolic cable at the support is given by,

$$\tan\phi_1 = \frac{4f}{l} = 4n. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

To find the length of the cable, L, use the general formula,

$$L = 2\int_0^{\frac{l}{2}} \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} \cdot dx. \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

Substituting the value of $\frac{dy}{dx}$ obtained from Eq. (13), we have,

$$L = 2\int^{\frac{l}{2}} \left[1+\frac{64f^2x^2}{l^4}\right]^{\frac{1}{2}} \cdot dx, \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

which yields, upon integration,

$$L = \frac{l}{2}(1+16n^2)^{\frac{1}{2}} + \frac{l}{8n}\log_e\left[4n+(1+16n^2)^{\frac{1}{2}}\right]. \quad . \qquad (20)$$

This formula gives the exact length of the parabola between two ends at equal elevation.

For more expeditious solution, when a good table of hyperbolic functions is available, Eq. (20) may be written in the form

$$L = \frac{l}{16n}(2u+\sinh 2u), \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20')$$

where u is defined by $\sinh u = 4n$.

An approximate formula for the length of curve may be obtained by expanding the binomial in Eq. (19) and then integrating. This gives,

$$L = l\left(1+\tfrac{8}{3}n^2 - \tfrac{32}{5}n^4 + \ . \ . \ . \ \right), \quad . \quad . \quad . \quad . \qquad (21)$$

where n is defined by Eq. (16). For small values of the sag-ratio n, it will be sufficiently accurate to write,

$$L = l(1 + \tfrac{8}{3}n^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

for the length of a parabolic cable in terms of its chord l.

The following table gives the values of L as computed by Eqs. (20) and (21), respectively.

Sag-Ratio $\left(n=\frac{f}{l}\right)$	Length-Ratio $=\frac{L}{l}$	
	Exact (Eq. 20)	Approximate (Eq. 21)
.05	1.00662	1.00663
.075	1.01475	1.01480
.1	1.02603	1.02603
.125	1.04019	1.04010
.15	1.05693	1.05676
.175	1.07647	1.07566
.2	1.09822	1.09643

3. Unsymmetrical Spans.—If the two ends of a cable span are not at the same elevation, the ordinates y should be measured vertically from the inclined closing chord AB (Fig. 2). If that is done, all of the principles derived above will remain applicable, and Eqs. (1) to (14), inclusive, may be kept unchanged. For a load uniform along the horizontal, the curve will be a parabola, and its equation, referred to the origin A and to the axis AB, will be as before,

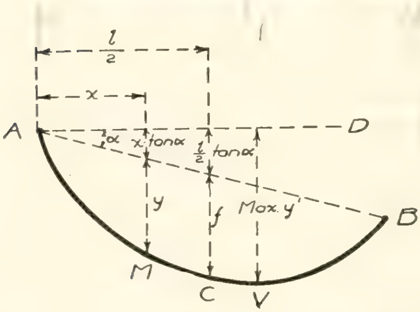

FIG. 2.—Unsymmetrical Parabolic Cable.

$$y = \frac{4fx}{l^2}(l - x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If it is desired to refer the curve to the horizontal line AD, with which the closing chord makes an angle α, the equation becomes,

$$y' = y + x \cdot \tan\alpha = \frac{4fx}{l^2}(l-x) + x \cdot \tan\alpha. \quad . \quad . \quad . \quad (23)$$

To find the lowest point in the curve, located at V, a little to one side of the center, differentiate Eq. (23) and place the result equal to zero. Solving for x, we obtain,

$$x_r = \frac{l}{2}\left(1 + \frac{l}{4f} \cdot \tan\alpha\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

To find the exact length of the curve, apply Eq. (20) to the segments VA and VB (Fig. 2), treating each of these segments as one-half of a complete parabola, and add the results.

An extreme case of the unsymmetrical parabolic curve occurs in the side-span cables of suspension bridges. Using the notation shown in Fig. 3, the equation of the curve may be written in the same way as Eq. (14),

$$y_1 = \frac{4f_1x_1}{l_1^2}(l_1 - x_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Here, again, y_1 and f_1 are measured vertically from the closing chord, and x_1 and l_1 are measured horizontally.

The true vertex of the curve or lowest point, V, will generally be found, by an equation similar to Eq. (24), to be outside point D (Fig. 3). The exact length of curve will be $VA - VD$, or the difference between two semi-parabolas each of which may be calculated by Eq. (20).

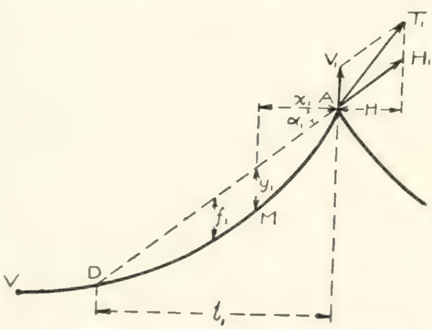

FIG. 3.—Parabolic Cable in Side Span.

An approximate value of the length may be obtained by

taking the closing chord, $AD = l_1 \cdot \sec\alpha_1$, and adding the parabolic curvature correction as in Eq. (22). This will yield

$$L_1 = l_1\left(\sec\alpha_1 + \frac{8}{3}\frac{n_1^2}{\sec^3\alpha_1}\right), \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where

$$n_1 = \frac{f_1}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The cable tension in the side span acts in the line of the closing chord AD (Fig. 3) and is designated by

$$H_1 = H \cdot \sec\alpha_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Since the lever arms y_1 are vertical, they must be multiplied by the horizontal component of H_1, or H, to obtain the bending moments produced by this force. Hence, as in Eq. (2), we have,

$$M' = H \cdot y_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and, as in Eq. (11), we obtain,

$$H = \frac{w_1 l_1^2}{8f_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

In order that the main and side spans may have equal values of H, by Eqs. (11) and (30), we have,

$$\frac{wl^2}{8f} = \frac{w_1 l_1^2}{8f_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Hence the necessary relation between the sags is

$$\frac{f_1}{f} = \frac{w_1 l_1^2}{wl^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The stress at any point in the cable is given by Eq. (5), which may be rewritten as

$$T = H(1 + \tan^2\phi)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

At the center of the side span, where $x_1 = \frac{l_1}{2}$, the curve is parallel

to the chord, and the inclination is equal to α_1; hence, at that point

$$T = H(1+\tan^2\alpha_1)^{\frac{1}{2}}. \qquad (34)$$

At the support, where $x_1=0$, the inclination of the cable is given by

$$\tan\phi_1 = \tan\alpha_1 + \frac{4f_1}{l_1}, \qquad (35)$$

and formula (33) yields

$$T_1 = H\left[1+\left(\tan\alpha_1+\frac{4f_1}{l_1}\right)^2\right]^{\frac{1}{2}}, \qquad (36)$$

which is the maximum stress in the cable.

4. The Catenary. If the load w is not constant per horizontal unit, but per unit length of the curve, as is the case where the load on the cable is due to its own weight, Eq. (10) takes the form,

$$\frac{d^2y}{dx^2} = -\frac{w\cdot\sec\phi}{H}. \qquad (37)$$

Since $\tan\phi=\frac{dy}{dx}$, Eq. (37) may be written,

$$\frac{d^2y}{dx^2} = -\frac{w}{H}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}. \qquad (38)$$

Integrating this equation, taking the origin at the lowest point of the curve, we obtain the equation of the cable curve:

$$y=\frac{1}{2c}(e^{cx}+e^{-cx}-2), \qquad (39)$$

where $c=\frac{w}{H}$.

This is the equation of a catenary; a cable under its own weight hangs in a catenary.

Replacing the exponential terms by hyperbolic functions, Eq. (39) may be written,

$$y=\frac{1}{c}(\cosh cx-1). \qquad (40)$$

To find the length of the catenary, substitute $\frac{dy}{dx}$ obtained from Eq. (39) in Eq. (18). This gives

$$L=2\int_0^{\frac{l}{2}}\tfrac{1}{2}(e^{cx}+e^{-cx})dx=\frac{1}{c}(e^{\frac{cl}{2}}-e^{-\frac{cl}{2}}). \quad . \quad . \quad . \quad (41)$$

Expressed in hyperbolic functions, Eq. (41) may be written,

$$L=\frac{2}{c}\sinh\frac{cl}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Equations (40) and (42) are useful in computations for the guide wires employed for the regulation of the strands in cable erection. If the length L is known, Eq. (42) may be solved for the parameter c, by a method of successive approximations, and the ordinates may then be obtained from Eq. (40). For the expeditious solution of these equations, good tables of hyperbolic functions are required.

If the integration in Eq. (41) is performed between the limits 0 and x, and the value of y substituted from Eq. (39), we obtain,

$$L=\frac{1}{c}\sqrt{2cy+c^2y^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

as a formula for the length from the vertex to any point of the curve. Equation (43) may be used for unsymmetrical catenaries.

The stress at any point in the cable is again given by Eq. (5), or,

$$T=H\cdot\frac{ds}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Since $H=\frac{w}{c}$, Eq. (44) may be written:

$$T=\frac{w}{c}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}.$$

Substituting the value of $\frac{dy}{dx}$ derived from Eq. (39), we obtain,

$$T=\frac{w}{2c}(e^{cx}+e^{-cx}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Replacing the exponential by hyperbolic functions, Eq. (45) becomes,

$$T = H \cdot \cosh cx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

This tension will be a maximum at the ends of the span, where $x = \frac{l}{2}$, yielding,

$$T_1 = H \cdot \cosh \frac{cl}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Comparing Eqs. (40) and (46), we find,

$$T = w\left(y + \frac{1}{c}\right) = wy + H \quad . \quad . \quad . \quad . \quad (48)$$

At the span center, where $y = 0$, this gives $T = H$; and at the supports, where $y = f$, we obtain,

$$T_1 = wf + H. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

If the sag-ratio $\left(n = \frac{f}{l}\right)$ is small, all of the formulas for the catenary may be replaced, with sufficient accuracy, by the formulas for parabolic cables.

5. Deformations of the Cable.—As a result of elastic elongation, slipping in the saddles, or temperature changes, the length of cable between supports may alter by an amount ΔL; as a result of tower deflection or saddle displacement, the span may alter by an amount Δl. Required to find the resulting changes in cable-sag, Δf.

For parabolic cables, the length is given with sufficient accuracy by Eq. (21). Partial differentiation of that equation with respect to l and f, respectively, yields the two relations:

$$\Delta L = \tfrac{1}{15}(15 - 40n^2 + 288n^4) \cdot \Delta l, \quad . \quad . \quad . \quad (50)$$

$$\Delta L = \tfrac{16}{15}n \cdot (5 - 24n^2) \cdot \Delta f. \quad . \quad . \quad . \quad . \quad . \quad (51)$$

From Eqs. (50) and (51), there results,

$$\Delta f = -\frac{15 - 40n^2 + 288n^4}{16n(5 - 24n^2)} \cdot \Delta l. \quad . \quad . \quad . \quad . \quad (52)$$

The required center deflections may be calculated by means of Eqs. (51) and (52) when ΔL and Δl are known.

For a change in temperature of t degrees, coefficient of expansion ω, the change in cable-length will be,

$$\Delta L = \omega \cdot t \cdot L. \qquad (53)$$

For any loading which produces a horizontal tension H, the average stress in the cable will be, very closely,

$$\frac{L}{l} \cdot H,$$

and the elastic elongation will be,

$$\Delta L = \frac{L}{l} \cdot \frac{HL}{EA}, \qquad (54)$$

where E is the coefficient of elasticity and A is the area of cross-section of the cable.

Another expression for the elastic elongation is

$$\Delta L = \frac{H}{EA} \int_0^L \frac{ds^2}{dx} = \frac{Hl}{EA}(1 + \tfrac{16}{3}n^2). \qquad (55)$$

For a small change in the cable-sag Δf, the resulting change in the horizontal tension is obtained by differentiating Eq. (12):

$$\Delta H = -\frac{H}{f} \cdot \Delta f. \qquad (56)$$

From Eqs. (56), (51) and (52), may be found the deformations of the cable produced by any small change in the cable stresses.

SECTION II.—UNSTIFFENED SUSPENSION BRIDGES

6. Introduction.—The unstiffened suspension bridge is not used for important structures. The usual form, as indicated in Fig. 4, consists of a cable passing over two towers and anchored by back-stays to a firm foundation. The roadway is suspended from the cable by means of hangers or suspenders. As there is no stiffening truss, the cable is free to assume the equilibrium curve of the applied loading.

7. Stresses in the Cables and Towers.—If built-up chains are used, as in the early suspension bridges, the cross-section may be varied in proportion to the stresses under maximum loading. In a wire cable, the cross-section is uniform throughout.

As the cable and hangers are light in comparison with the roadway, the combined weight of the three may be considered as uniformly distributed along the horizontal. Let this total dead load be w pounds per lineal foot. The cable will then assume a parabolic curve; and all of the relations derived for a parabolic cable, represented by Eqs. (11) to (22), will apply.

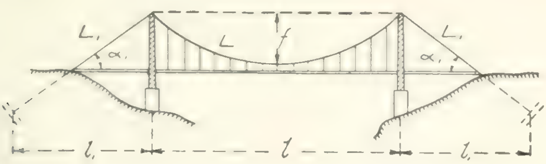

FIG. 4.—Unstiffened Suspension Bridge.

The maximum dead-load stress in the cable, occurring at the towers, is given by Eq. (15):

$$T_w = \frac{wl^2}{8f}(1+16n^2)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where n is the ratio of the sag f to the span l.

Let there be a uniform live load of p pounds per lineal foot. The maximum cable stress will evidently occur when the load covers the whole span, and will have a value,

$$T_p = \frac{pl^2}{8f}(1+16n^2)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Adding the values in Eqs. (15) and (57), we find the total stress in the cable at the towers:

$$T_{w+p} = \frac{(w+p)l^2}{8f}(1+16n^2)^{\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad . \quad (58)$$

If α_1 is the inclination of the backstay to the horizontal (Fig. 4), the stress in the backstay will be:

$$T_1 = H \cdot \sec \alpha_1 = \frac{(w+p)l^2}{8f} \cdot \sec \alpha_1 \quad . \quad . \quad (59)$$

If cable and backstay have equal inclinations at the tower, their stresses, represented by Eqs. (58) and (59), will be equal.

The vertical reaction of the main cable at the tower is $(w+p)l/2$. If the backstay has the same inclination as the cable, it will also have the same vertical reaction; so that the total stress in the tower will be,

$$T = (w+p) \cdot l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

8. Deformations under Central Loading.—Under partial loading, the unstiffened cable will be distorted from its initial parabolic curve. It is required to find the deflections produced by the change of curve, disregarding for the present any stretching of the cable or any displacement of the saddles.

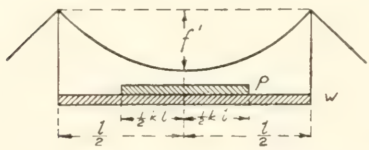

FIG. 5.—Loading for Maximum Vertical Deflection.

The maximum vertical deflection at the center of the cable will occur when a certain central portion of length kl is covered with live load (p), in addition to the dead load (w) covering the whole span (Fig. 5). The sag of the distorted cable will be, by Eq. (3),

$$f' = \frac{wl^2}{8H} + \frac{pl^2}{8H} \cdot k(2-k). \quad . \quad . \quad . \quad . \quad (61)$$

Equating the expressions for the cable-length corresponding to the initial and distorted conditions, respectively, the lengths being obtained from the approximate equation (22), and introducing the symbol $q = p/w$, we obtain:

$$L = l(1 + \tfrac{8}{3}n^2) = l + \frac{w^2 l^3}{24H^2}(1 + 3qk + 3q^2k^2 - qk^3 - 2q^2k^3). \quad (62)$$

Solving this equation for H, and substituting in Eq. (61), there results:

$$\frac{f'}{f}=\frac{1+2qk-qk^2}{(1+3qk+3q^2k^2-qk^3-2q^2k^3)^{\frac{1}{2}}}. \quad . \quad . \quad (63)$$

By differentiating this expression with respect to k, we obtain the following condition for a maximum value of f':

$$k^4(1+2q)q+2k^3(1-q)q+3k^2(1-q)-4k+1=0. \quad . \quad (64)$$

Solving this equation for k and substituting the result in Eq. (63), we obtain the following values for the maximum crown deflection $\Delta f=f'-f$:

For $q=\frac{p}{w}=$	0	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
$k=$	1.0	0.64	0.30	0.28	0.25	0.23	0.21
$\Delta f=$	0	.013	.022	.028	.045	.067	.080f

From this tabulation we may obtain the following empirical values, sufficiently accurate between the limits $q=\frac{p}{w}=\frac{1}{4}$ to 4:

$$\left.\begin{aligned} k&=0.20+\frac{0.05}{q} \\ \Delta f&=(0.007+0.046q-0.0075q^2)f \end{aligned}\right\} \quad . \quad . \quad . \quad (65)$$

9. Deformations under Unsymmetrical Loading. The greatest distortion of the cable from symmetry, represented by the maximum horizontal displacement of the low point or vertex, will be produced by a continuous uniform load extending for some distance kl from the end of the span. (Fig. 6.)

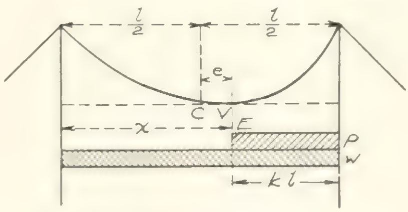

FIG. 6.—Maximum Horizontal Displacement of the Crown.

Applying the principle of Eq. (3), the lowest point of the cable curve is located by the condition $\frac{dM'}{dx}=0$; accordingly,

with the notation of Fig. 6, so long as the crown V is to the left of the head of the load (E),

$$\frac{dM'}{dx}=\frac{w}{2}(l-2x)+\tfrac{1}{2}pk^2l=0. \quad . \quad . \quad . \quad . \quad (66)$$

Inspection of this equation shows that x will have its maximum value when k has its maximum value; that is, when $kl=l-x$; in other words, the greatest lateral displacement occurs when the head of the moving load reaches the low point, V. Substituting this value in Eq. (66), we obtain:

$$k=-\frac{w}{p}+\sqrt{\frac{w}{p}+\frac{w^2}{p^2}}. \quad . \quad . \quad . \quad . \quad . \quad (67)$$

Hence the maximum deviation of the crown (V) from the center of the span (c), will be (Fig. 6),

$$\frac{c}{l}=\frac{1}{2}+\frac{w}{p}-\sqrt{\frac{w}{p}+\frac{w^2}{p^2}}. \quad . \quad . \quad . \quad . \quad . \quad (68)$$

The total sag of the cable is practically invariable for all ordinary values of p/w. Consequently, the uplift of the cable at the center of the span will amount to ,

$$\Delta f=\left(\frac{2c}{l+2c}\right)^2\cdot f. \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

We thus obtain the following values:

For $\frac{p}{w}=$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{4}{3}$	2	3	4
$c=$	.028	.036	.051	.086	.105	.134	.167	.191l
$\Delta f=$	.003	.004	.008	.021	.030	.045	.062	.076f

10. Deflections Due to Elongation of Cable.—The total length of cable, including the backstays (Fig. 4), is, by Eq. (21),

$$L+2L_1=l(1+\tfrac{8}{3}n^2-\tfrac{32}{5}n^4)+2l_1\sec\alpha_1. \quad . \quad . \quad . \quad (70)$$

For a change in temperature of t degrees, the total elongation of cable will be

$$\Delta L=\omega\cdot t(L+2L_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

For the elongation of the cable due to elastic strain, we may write, by Eq. (55),

$$\Delta L=\frac{H}{EA}[l(1+\tfrac{16}{3}n^2)+2l_1\cdot\sec^2\alpha_1]. \quad . \quad . \quad (72)$$

In addition there may be a contribution to ΔL from yielding of the anchorages.

If the cable is capable of slipping over the fixed saddles, the resulting deflection Δf is obtained by substituting the above values of ΔL in Eq. (51).

If, however, a displacement of the saddles or a movement of the tops of the towers will occur before the cable will slip, any elongation of the backstays will alter the span (l), but not the length (L), of the cable in the main span. In that case, the combined effects of temperature and elastic strain will give:

$$\Delta L=\omega tL+\frac{Hl}{EA}(1+\tfrac{16}{3}n^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

and

$$\Delta l=-2\sec\alpha_1\cdot\left(\omega tl_1\cdot\sec\alpha_1+\frac{Hl_1}{EA_1}\cdot\sec^2\alpha_1\right). \quad . \quad . \quad (74)$$

Substituting these values of ΔL and Δl in Eqs. (51) and (52), respectively, we obtain the resulting deflection (Δf) of the main cable:

$$\Delta f=\frac{15}{16(5n-24n^3)}\cdot\Delta L-\frac{15-40n^2+288n^4}{16(5n-24n^3)}\cdot\Delta l. \quad . \quad (75)$$

If a displacement of the saddles (Δl) is accompanied by a slipping of the cable, so that the total length of the latter between anchorages (Fig. 4) remains unchanged, then the changes in length and span of the main cable must satisfy the relation

$$\Delta L=\Delta l\cdot\cos\alpha_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Substituting these values in Eq. (75), the crown deflection becomes,

$$\Delta f=\frac{15\cdot\cos\alpha_1-(15-40n^2+288n^4)}{16(5n-24n^3)}\cdot\Delta l. \quad . \quad . \quad (77)$$

SECTION III.—STIFFENED SUSPENSION BRIDGES

11. Introduction.—In order to restrict the static distortions of the flexible cable discussed in the preceding pages, there is introduced a stiffening truss connected to the cable by hangers (Figs. 7, 15, 16). The side spans may likewise be suspended from the cable (Figs. 10, 11, 18), or they may be independently supported; in the latter case the backstays will be straight (Figs. 15, 16, 20). The main-span truss may be simply supported at the towers (Figs. 11, 16), or it may be built continuous with the side spans (Figs. 18, 20). A hinge may be introduced at the center of the stiffening truss in order to make the structure statically determinate (Fig. 8), or to reduce the degree of indeterminateness.

Another form of stiffened suspension bridge is the braced-chain type. This type does not make use of the straight stiffening truss suspended from a cable; instead, the suspension system itself is made rigid enough to resist distortion, being built in the form of an inverted arch (Figs. 21, 22, 23, 24).

For ease of designation, it will be convenient to adopt a symbolic classification of stiffened suspension bridges, based on the number of hinges in the main span of the truss, as tabulated on page 19.

In types $2F$ and $3F$, the side spans are not related to the main elements of the structure and may therefore be omitted from consideration. Hence these types are called "single-span bridges."

The suspension bridges with straight stiffening trusses will be analyzed first.

12. Assumptions Used.—In the theory that follows, we adopt the assumption that the truss is sufficiently stiff to render the deformations of the cable due to moving load practically negligible; in other words, we assume, as in all other rigid structures, that the lever arms of the applied forces are not altered by the deformations of the system. The resulting theory is the one ordinarily employed, and is sufficiently accurate for all practical purposes; any errors are generally small and on the side of safety.

Stiffened Suspension Bridges

- *Stiffening truss*
  - *Continuous*
    - Side span free $=0F$ (Fig. 20)
    - Side span suspended $=0S$ (Fig. 18)
  - *One-hinged*
    - Side span free $=1F$
    - Side span suspended $=1S$
  - *Two-hinged*
    - Side span free $=2F$ (Fig. 16)
    - Side span suspended $=2S$ (Fig. 11)
  - *Three-hinged*
    - Side span free $=3F$ (Fig. 8)
    - Side span suspended $=3S$ (Fig. 26)
- *Braced chain*
  - *Hingeless* $=0B$ (Fig. 24)
  - *One-hinged* $=1B$
  - *Two-hinged* $=2B$ (Fig. 23)
  - *Three-hinged* $=3B$ (Figs. 21, 22)

If the stiffening truss is not very stiff or if the span is long, the deflections of truss and cable may be too large to neglect. To provide for such cases, there has been developed an exact method of calculation which takes into account the deformations of the system. For lack of space, this "Exact Theory" will not be presented here, but the interested reader is referred instead to other works on the subject.*

The common theory developed in the following pages for the analysis of suspension bridges with stiffening trusses is based on five assumptions, which are very near the actual conditions:

1. The cable is supposed perfectly flexible, freely assuming the form of the equilibrium polygon of the suspender forces.

2. The truss is considered a beam, initially straight and

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* Melan-Steinman: "Theory of Arches and Suspension Bridges," pp. 76–86. McGraw-Hill Book Co. 1913.

Johnson, Bryan and Turneaure: "Modern Framed Structures," Part II, pp. 276–318. John Wiley & Sons. 1911.

Burr, W. H.: "Suspension Bridges," pp. 212–247. John Wiley & Sons. 1913.

horizontal, of constant moment of inertia and tied to the cable throughout its length.

3. The dead load of truss and cable is assumed uniform per lineal unit, so that the initial curve of the cable is a parabola.

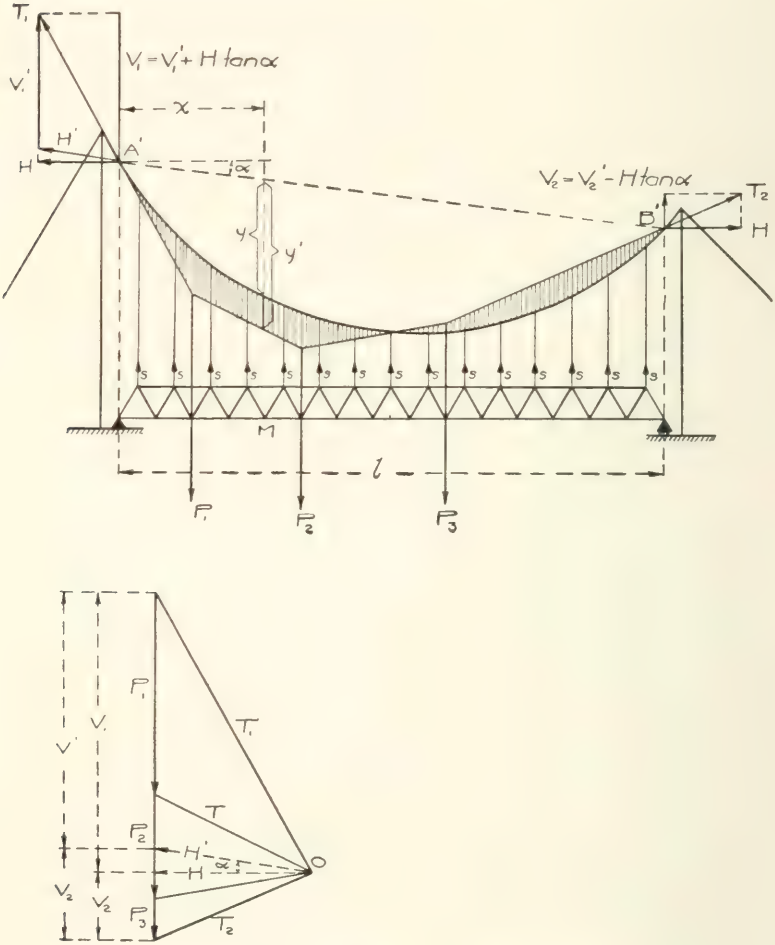


FIG. 7.—Forces Acting on the Stiffening Truss.

4. The form and ordinates of the cable curve are assumed to remain unaltered upon application of loading.

5. The dead load is carried wholly by the cable and causes

no stress in the stiffening truss. The truss is stressed only by live load and by changes of temperature.

The last assumption is based on erection adjustments, involving regulation of the hangers and riveting-up of the trusses when assumed conditions of dead load and temperature are realized.

13. Fundamental Relations.—Since the cable in the stiffened suspension bridge is assumed to be parabolic, the loads acting on it must always be uniform per horizontal unit of length. All of the relations established for a uniformly loaded cable (Eqs. (11) to (36), inclusive) will apply in this case.

If the panel points are uniformly spaced (horizontally), the suspender forces must be uniform throughout (Fig. 7). These suspender forces are loads acting downward on the cable, and upward on the stiffening truss. It is the function of the stiffening truss to take any live load that may be arbitrarily placed upon it and distribute it uniformly to the hangers.

The cable maintains equilibrium between the horizontal tension H (resisted by the anchorages) and the downward acting suspender forces. If these suspender forces per horizontal linear unit are denoted by s, they are given by Eq. (11) as

$$s=H\cdot\frac{8f}{l^2}. \qquad (78)$$

The truss (Fig. 7) must remain in equilibrium under the arbitrarily applied loads acting downward and the uniformly distributed suspender forces acting upward. If we imagine the latter forces removed, then the bending moment M' and the shear V' at any section of the truss, distant x from the left end, may be determined exactly as for an ordinary beam (simple or continuous according as the truss rests on two or more supports). This moment and shear would be produced if the cable did not exist and the entire load were carried by the truss alone. If $-M_s$ represents the bending moment of the suspender forces at the section considered, then the total moment in the stiffening truss will be

$$M=M'-M_s. \qquad (79)$$

Similarly, if $-V_s$ represents the shear produced by the suspender forces at the same section, the total shear in the stiffening truss will be

$$V = V' - V_s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

Equations (79) and (80) are the fundamental formulas for determining the stresses in any stiffening truss. By these formulas, the stresses can be calculated for any given loading as soon as the value of H is known.

The dead load is assumed to be exactly balanced by the initial suspender forces, so that it may be omitted from consideration in these equations.

In calculating M' and V' from the specified live load, and M_s and V_s from the uniform suspender loading given by Eq. (78), the condition of the stiffening truss as simple or continuous must be taken into account.

If the stiffening truss is a simple beam (hinged at the towers), by a familiar property of the funicular polygon, represented by Eq. (2),

$$M_s = H \cdot y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

where y is the ordinate to the cable curve measured from the straight line joining A' and B', the points of the cable directly above the ends of the truss (Fig. 7). Consequently, Eq. (79) may be written,

$$M = M' - H \cdot y. \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

which is identical with equation (1). (In the unstiffened suspension bridge, $M = 0$.)

If ϕ is the inclination of the cable at the section considered, the shear produced by the hanger forces is given by Eq. (7) as,

$$V_s = H \cdot \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Consequently, Eq. (80) may be written

$$V = V' - H \cdot \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

If the two ends of the cable, A' and B', are at unequal elevations (Fig. 7), Eq. (84) must be corrected to the form,

$$V = V' - H\,(\tan \phi - \tan \alpha), \quad . \quad (84')$$

where α is the inclination of the closing line $A'B'$ below the horizontal.

In Eqs. (82), (84) and (84′), the last term represents the relief of bending moment or shear by the cable tension H.

Representing M' by the ordinates y' of an equilibrium polygon or curve, constructed for the applied loading with a pole distance $=H$, Eq. (82) takes the form,

$$M=H(y'-y). \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

Hence the bending moment at any section of the stiffening truss is represented by the vertical intercept between the axis of the cable and the equilibrium polygon for the applied loads drawn through the points $A'B'$ (Fig. 7).

If the stiffening truss is continuous over several spans, the relations represented by Eqs. (81) to (85), inclusive, must be modified to take into account the continuity at the towers. The corresponding formulas will be developed in the section on continuous stiffening trusses (Section VI).

14. Influence Lines.—To facilitate the study and determination of suspension bridge stresses for various loadings, influence diagrams are most convenient.

The base for all influence diagrams is the H-curve or H-influence line. This is obtained by plotting the equations giving the values of H for varying positions of a unit concentration. In the case of three-hinged suspension bridges, the H-influence line is a triangle (Figs. 8 and 9). In the case of two-hinged stiffening trusses, the H-lines (Figs. 11, 14) are similar to the deflection curves of simple beams under uniformly distributed load. In the case of continuous stiffening trusses, the H-line (Fig. 18) is similar to the deflection curve of a three-span continuous beam covered with uniform load in the suspended spans.

To obtain the influence diagrams for bending moments and shears, all that is necessary is to superimpose on the H-curve, as a base, appropriately scaled influence lines for moments and shears in straight beams.

The general expression for bending moments at any section (Eq. 82) may be written in the form,

$$M = y\left(\frac{M'}{y} - H\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

(excepting that in the case of continuous stiffening trusses, y is to be replaced by $y-ef$; see Eq. 212). For a moving concentration, $\frac{M'}{y}$ represents the moment influence line of a straight beam, simple or continuous as the case may be, constructed with the pole distance y. Hence the moment M is proportional to the difference between the ordinates of this influence line and those of the H-influence line. If the two influence lines are superimposed (Figs. 8*b*, 11*b*, 11*c*, 18*b*), the intercepts between them will represent the desired bending moment M. In the case of stiffening trusses with hinges at the towers, M' is the same as the simple-beam bending moment, and its influence line is familiarly obtained as a triangle whose altitude at the given section is,

$$\frac{M'}{y} = \frac{x(l-x)}{l \cdot y}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

For a parabolic cable, this reduces (by Eq. 14) to

$$\frac{M'}{y} = \frac{l}{4f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

Hence the $\frac{M'}{y}$ triangles for all sections will have the same altitude $\frac{l}{4f}$ (Figs. 8*b*, 11*b*). The corresponding altitude for sections in the side spans is $\frac{l_1}{4f_1}$ (Fig. 11*c*). The areas intercepted between the H-line and the $\frac{M'}{y}$ triangles, multiplied by py, give the maximum and minimum bending moments at the given section, X, of the stiffening truss. Areas below the H-line represent positive moments, and those above represent negative moments

(Figs. 8, 11, 18). Where the two superimposed lines intersect, we have a point K, which may be called the zero point, since a concentration placed at K produces zero bending stress at X. K is also called the critical point, since it determines the limit of loading for maximum positive or negative moment at X. Load to one side of K yields plus bending, and load to the other side produces negative bending.

The shear at any section of the stiffening truss is given by Eq. (84), which may be written in the form,

$$V=\left(\frac{V'}{\tan\phi}-H\right)\cdot\tan\phi. \quad . \quad . \quad . \quad . \quad . \quad (89)$$

(If the two ends of the cable span are at different elevations, $\tan\phi$ in this equation is to be replaced by $\tan\phi-\tan\alpha$, where α is the inclination of the closing chord below the horizontal. See Eq. 84′). For any given section X, $\tan\phi$, the slope of the cable, is a constant and is given by,

$$\tan\phi=\frac{4f}{l}\left(1-\frac{2x}{l}\right)+\tan\alpha \quad . \quad . \quad . \quad . \quad (90)$$

The values assumed by the bracketed expression in Eq. (89) for different positions of a concentrated load may be represented as the difference between the ordinates of the H-line and those of the influence line for the shears V', the latter being reduced in the ratio $\dfrac{1}{\tan\phi}$. The latter influence line is familiarly obtained by drawing the two parallel lines as and bt (Figs. 9*a*, 9*b*, 14*a*), their direction being fixed by the end intercepts

$$am=bn=\frac{1}{\tan\phi-\tan\alpha}=\frac{l}{4f\left(1-2\dfrac{x}{l}\right)}. \quad . \quad . \quad (91)$$

The vertices s and t lie on the vertical passing through the given section X. The maximum shears produced by a uniformly distributed load are determined by the areas included between the H and V' influence lines; all areas below the H-line are to be considered positive, and all above negative. These areas must

be multiplied by $p \cdot \tan\phi$ (or by $p[\tan\phi - \tan\alpha]$) to obtain the greatest shear V at the section; and V must be multiplied by the secant of inclination to get the greatest stress in the web members cut by the section.

SECTION IV.—THREE-HINGED STIFFENING TRUSSES

15. Analysis.—This is the only type of stiffened suspension bridge that is statically determinate (Types $3F$, $3S$, $3B$). The provision of the stiffening truss with a central hinge furnishes a condition which enables H to be directly determined; viz., at the section through the hinge the moment M must equal zero. Consequently, if the bending moment at the same section of a simple beam is denoted by M'_0, and if f is the ordinate of the corresponding point of the cable, by Eq. (82),

$$H = \frac{M'_0}{f}. \qquad (92)$$

Hence the value of H for any loading is equal to the simple-beam bending moment at the center hinge divided by the sag f. Accordingly, the cable will receive its maximum stress when the full span is covered with the live load p. In that case Eq. (92) yields

$$H = \frac{pl^2}{8f}, \qquad (93)$$

and, comparing this with Eq. (78), we see that

$$s = p. \qquad (94)$$

Hence, under full live load, the conditions are similar to those for dead load, the cable carrying all the load, the trusses having no stress. The bending moment at any section will be

$$\text{Total } M = 0. \qquad (94')$$

For a single load P at a distance kl from the near end of the span, the simple-beam moment at the center hinge will be

$$M'_0 = \frac{Pkl}{2}. \qquad (95)$$

Hence, the value of H, by Eq. (92), will be

$$H = \frac{Pkl}{2f}. \qquad (96)$$

This value of H will be a maximum for $k = \frac{1}{2}$, yielding,

$$\text{Max. } H = \frac{Pl}{4f}. \qquad (97)$$

According to Eq. (92), the influence line for H will be similar to the influence line for bending moment M'_0 at the center of a simple beam; hence it will be a triangle. It is defined by Eq. (96); and its maximum ordinate (at the center of the span) is given by Eq. (97) as $l/4f$. Figures 8*b* and 9*a* show the H-influence line constructed in this manner.

If the truss is uniformly loaded for a distance kl from one end, the value of H may be found by integrating Eq. (96) or directly from Eq. (92). We thus obtain:

$$\text{for } k < \tfrac{1}{2}, \quad H = \frac{pl^2}{4f} \cdot (k^2), \qquad (98)$$

$$\text{for } k > \tfrac{1}{2}, \quad H = \frac{pl^2}{8f}(4k - 2k^2 - 1). \qquad (99)$$

For full load ($k = 1$), Eq. (99) gives the maximum value of H:

$$H = \frac{pl^2}{8f}. \qquad (100)$$

which is identical with Eq. (93). Equations (98) to (100), inclusive, may also be obtained directly from the H-influence line (Figs. 8*b* and 9*a*).

For the half-span loaded, Eqs. (98) and (99) yield,

$$H = \frac{pl^2}{16f}, \qquad (101)$$

which is one-half of the value for full load. Substituting this value in Eq. (78), we find,

$$s = \tfrac{1}{2}p. \qquad (102)$$

One-half of the span is thus subjected to an unbalanced upward load, $s = \frac{1}{2}p$, per lineal foot, and the other half sustains an equal

downward load, $p-s=\frac{1}{2}p$. Consequently there will be produced positive moments in the loaded half, and equal negative moments in the unloaded half, amounting to

$$M=\tfrac{1}{4}px\left(\frac{l}{2}-x\right); \quad . \quad . \quad . \quad . \quad . \quad . \qquad (103)$$

and the maximum moments for this loading, occurring at the quarter points, $(x=\frac{1}{4}l,\ x=\frac{3}{4}l)$, will be,

$$\mathrm{M}=\pm\tfrac{1}{64}pl^2=\pm 0.01562pl^2. \quad . \quad . \quad . \quad . \qquad (104)$$

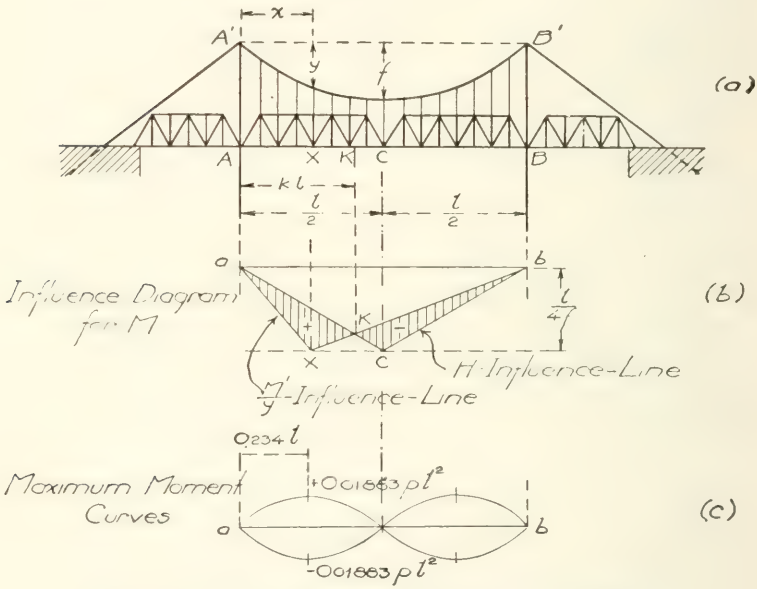


FIG. 8.—Three-hinged Stiffening Truss Moment Diagrams. (Type 3*F*).

16. Moments in the Stiffening Truss.—The influence diagrams for bending moments are constructed, in accordance with Eq. (86), by superimposing the $\dfrac{M'}{y}$ triangles upon the H-influence triangle. By Eq. (88), the $\dfrac{M'}{y}$ triangles for all sections have the same altitude $\dfrac{l}{4f}$; and, in the case of the three-hinged stiffening

truss, this altitude is identical with that of the H-influence triangle.

The two triangles are shown superimposed in Fig. 8b. The shaded area between them is the influence diagram for bending moment at the section X.

For $x=\frac{l}{2}$, the two triangles would coincide. Hence the moment at the center hinge is zero for all conditions of loading, which agrees with the condition that the hinge can carry no bending.

For $x<\frac{l}{2}$, the two influence triangles intersect at a point K, a short distance to the left of the center. K is the zero point or critical point. All load to the left of K yields plus bending, and all load to the right produces negative bending.

Since the two superimposed triangles have the same base and equal altitudes, the plus and minus intercepted areas will be equal. Hence, if the whole span is loaded, the two areas will cancel each other, yielding zero moment as required by Eq. (94′).

If either of the shaded areas is multiplied by py, it will give the maximum value of the bending moment at X. The bending moments may also be obtained analytically from Eq. (82), as follows:

If the load covers a length kl from one end of the span, the bending moment at any section $x<kl$, by Eqs. (82), (98) and (14), will be,

$$M=\tfrac{1}{2}pkl(2-3k)x-\tfrac{1}{2}px^2(1-2k^2). \quad . \quad . \quad (105)$$

Setting $\frac{dM}{dk}=0$ in this equation, we find that for maximum M,

$$k=\frac{l}{3l-2x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

This equation defines the distance kl to the critical point K (Fig. 8b). For this value of k, Eq. (105) gives the maximum value of M for any value of x:

$$\text{Max. } M=\frac{px(l-x)(l-2x)}{2(3l-2x)}. \quad . \quad . \quad . \quad (107)$$

This value may also be obtained from the shaded areas in the influence diagram (Fig. 8*b*). Setting $\frac{dM}{dx}=0$ in the last equation, we find that the absolute maximum M occurs at,

$$x=0.234l. \qquad \ldots \ldots \quad (108)$$

Substituting this value in Eqs. (107) and (106), we find that the absolute maximum value of M is,

$$\text{Abs. Max. } M=+0.01883pl^2, \quad \ldots \quad (109)$$

or about $\frac{1}{53}pl^2$, and that it occurs at $x=0.234l$, when $k=0.395$.

By loading the remainder of the span ($0.605l$), we obtain the maximum negative moment at the same section. This will be numerically equal to the maximum positive moment, since their summation at any section must give zero according to Eq. (94′). Hence the absolute maximum negative moment will be,

$$\text{Abs. Min. } M=-0.01883pl^2. \quad \ldots \quad (109')$$

After the maximum moments at the different sections along the span are evaluated from the influence lines, or from Eq. (107), they may be plotted in the form of curves, as shown in Fig. 8*c*. For the three-hinged stiffening truss, these maximum moment curves are symmetrical about the horizontal axis. They may be used as a guide for proportioning the chord sections of the stiffening truss.

17. Shears in the Stiffening Truss.—The shears produced in the stiffening truss by any loading are given by Eq. (84); but the maximum values at the different sections are most conveniently determined with the aid of influence lines (Fig. 9).

The influence line for H is a triangle, with altitude $=\frac{l}{4f}$ at the center of the span. Upon this is superimposed the influence line for shears in a simple beam, reduced in the ratio $1 : \tan \phi$. The resulting influence diagram for shear V at a given section $x<\frac{l}{4}$ is shown in Fig. 9*a*. There are two zero points or critical points: at x and at kl. The portion of the left span between

these two points must be loaded to produce maximum positive shear at the given section. From the geometry of the figure we find the position of the critical point K to be given by,

$$k = \frac{1}{3-4\frac{x}{l}}. \qquad (110)$$

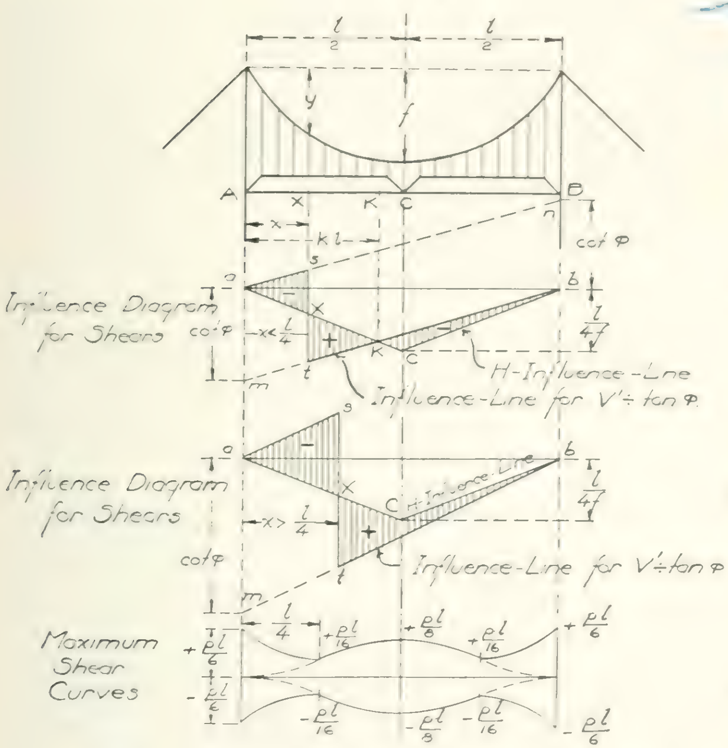


FIG. 9.—Shear Diagrams for Three-hinged Stiffening Truss. (Type 3F).

With the load covering the length from x to kl, we find the maximum positive shear at x, either from the diagram or from Eq. (84), to be given by

$$\text{Max. } V = \frac{pl}{2} \cdot \frac{\left(1-3\frac{x}{l}+4\frac{x^2}{l^2}\right)^2}{3-4\frac{x}{l}}. \qquad (111)$$

When $x=0$, or for end-shear, Eq. (110) gives $k=\frac{1}{3}$, and Eq. (111) yields,

$$\text{Abs. Max. } V=\frac{pl}{6}. \quad \ldots \ldots \quad (112)$$

When $x=\frac{1}{4}l$, we find $k=\frac{1}{2}$, and

$$\text{Max. } V=\frac{pl}{16}. \quad \ldots \ldots \quad (113)$$

For $x>\frac{1}{4}l$, the influence diagram takes the form shown in Fig. 9*b*. There is only one zero point, namely at the section X. Loading all of the span beyond X, we find the maximum positive shear, either from the diagram or from Eq. (84), to be given by,

$$\text{Max. } V=\frac{px^2}{2l}\left(3-4\frac{x}{l}\right). \quad \ldots \ldots \quad (114)$$

This has its greatest value for $x=\frac{1}{2}l$, or at the center, where it has the value,

$$\text{Max. } V=\frac{pl}{8}. \quad \ldots \ldots \quad (115)$$

Writing expressions for the maximum negative shears in the same manner, we obtain values identical with Eqs. (111) to (115), but with opposite sign. In other words, the plus and minus areas in any shear influence diagram are equal; their algebraic sum must be zero, since full span loading produces no stress in the stiffening truss. (See Eq. 94).

Figure 9*c* gives curves showing the variation of maximum positive and negative shears from end to end of the span. The curves are a guide for the proportioning of the web members of the stiffening truss. For the three-hinged truss, these curves are symmetrical about the horizontal axis.

If the two ends of the cable are at unequal elevations, the foregoing results for shear (Eqs. (110) to (115), inclusive) must be modified on account of the necessary substitution throughout the analysis of $(\tan\phi-\tan\alpha)$ for $\tan\phi$ as required by Eq. (84′).

SECTION V.—TWO-HINGED STIFFENING TRUSSES

18. Determination of the Horizontal Tension H.—In these bridge systems, the horizontal tension H is statically indeterminate. The required equation for the determination of H must therefore be deduced from the elastic deformations of the system.

Imagine the cable to be cut at a section close to one of the anchorages. Then (with $H=0$), under the action of any loads applied on the bridge, the two cut ends would separate by some horizontal distance Δ. If a unit horizontal force ($H=1$) be applied between the cut ends, it would pull them back toward each other a small distance δ. The total horizontal tension H required to bring the two ends together again would evidently be the ratio of these two imaginary displacements, or,

$$H=\frac{\Delta}{\delta}. \qquad (116)$$

Substituting for Δ and δ the general expressions for the displacement of a point

FIG. 10.—Philadelphia–Camden Bridge. Span 1750 Feet. (Type 2S).
Design by Delaware River Bridge Commission. 1921.

in an elastic system under the action of given forces, Eq. (116) becomes,

$$H=\frac{\Delta}{\delta}=-\frac{\int\frac{M'm}{EI}dx}{\int\frac{m^2}{EI}\cdot dx+\int\frac{u^2}{EA}\cdot ds}, \quad . \quad . \quad . \quad (117)$$

where M' = bending moments (in the stiffening truss) under given loads, for $H=0$.

m = bending moments (in the stiffening truss) with zero loading, for $H=1$.

u = direct stresses (in the cable, towers and hangers) with zero loading, for $H=1$.

I = moments of inertia (of the stiffening truss).

A = areas of cross-section (of the cable, towers and hangers).

In the denominator of Eq. (117) there are two terms, since the system is made up of members, part of which are acted upon by bending moments, and part by direct, or axial, stresses. In the numerator, there is no axial stress term, since for the condition of loading producing Δ, the cable tension $H=0$, and all of the axial stresses (in cables, towers and hangers) vanish.

Equation (117) is the most general form of the expression for H, and applies to any type of stiffened suspension bridge.

When there are no loads on the span, the bending moments in the two-hinged stiffening truss are, by Eq. (82):

$$M=-H\cdot y. \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

Hence we have, when $H=1$,

$$m=-y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

The stress at any section of the cable is given by Eq. (5), which, for $H=1$, reduces to,

$$u=\frac{ds}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Substituting Eqs. (119) and (120) in Eq. (117), we obtain the following fundamental equation for H for two-hinged stiffening trusses (Types $2F$ and $2S$):

$$H=\frac{\Delta}{\delta}=\frac{\int \frac{M'y}{EI}dx}{\int \frac{y^2dx}{EI}+\int \frac{ds^3}{EAdx^2}}. \quad . \quad . \quad . \quad . \quad (121)$$

The integral in the numerator and the first integral in the denominator represent the contributions of the bending of the stiffening truss to Δ and δ respectively; the integrations extend over the full length of the stiffening truss suspended from the cable. The second integral in the denominator represents the contribution of the cable stretch to the value of δ; the integration extends over the full length of cable between anchorages.

In the denominator of Eq. (121), the truss term contributes about 95 per cent, and the cable term only about 5 per cent of the total. Hence, certain approximations are permissible in evaluating the cable term.

Terms for the towers and hangers have been omitted, as they are negligible. (Their contribution to the denominator would be only a small decimal of 1 per cent.)

The terms in the denominator are independent of the loading and will now be evaluated. See Fig. 11a for the notation employed. The symbols l, x, y, f, α, I, A have already been defined for the main span; and, adding a subscript, we have the corresponding symbols l_1, x_1, y_1, f_1, α_1, I_1, A_1, for the side spans. In addition we have,

l' = span of the cable, center to center of towers, which distance may be somewhat greater than the truss span l (Fig. 11a);

l_2 = horizontal distance from tower to anchorage, which distance may be greater than the truss span l_1 (Fig. 11a).

Substituting for y its values from Eqs. (14) and (25), the

first integral in the denominator of Eq. (121), extending over main and side spans, becomes,

$$\int \frac{y^2 dx}{EI} = \frac{1}{EI}\int^{l} y^2 dx + 2\cdot\frac{1}{EI}\int_0^{l_1} y_1^2 dx = \frac{8}{15}\frac{f^2 l}{EI} + 2\left(\frac{8}{15}\frac{f_1^2 l_1}{EI_1}\right). \quad (122)$$

The moments of inertia I and I_1 are here assumed constant over the respective spans.

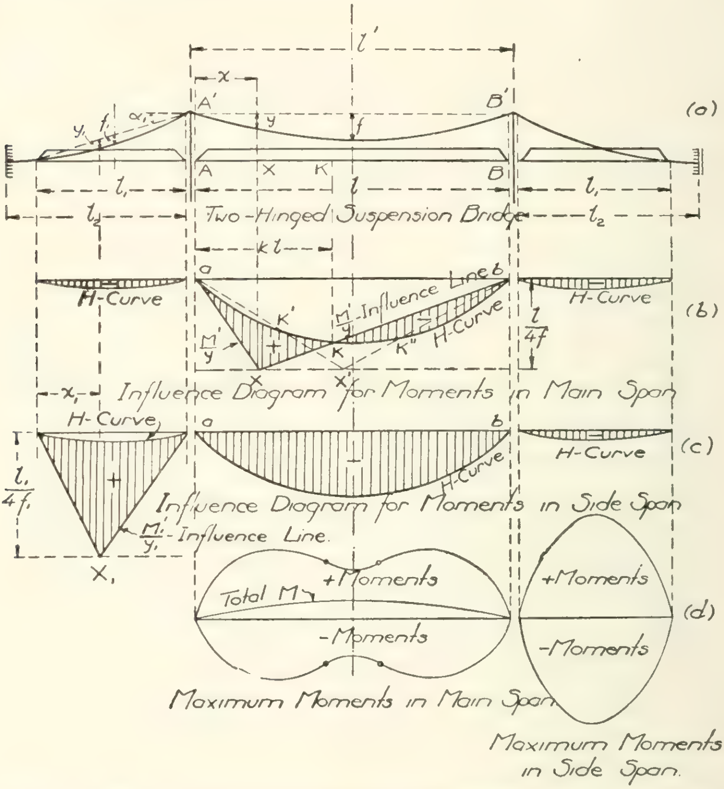


FIG. 11.—Moment Diagrams for Two-hinged Stiffening Truss. (Type 2S).

The second integral in the denominator of Eq. (121), with the aid of Eqs. (13) and (23), may be written,

$$\int \frac{ds^3}{EA\cdot dx^2} = \frac{2}{EA}\int_0^{\frac{l'}{2}} \left(1 + 64\frac{f^2x^2}{l^4}\right)^{3/2} dx$$

$$+ \frac{2}{EA_1}\cdot\int_0^{l_2}\left[1 + \left(\frac{4f_1}{l_1} - \frac{8f_1x_1}{l_1^2} + \tan\alpha_1\right)^2\right]^{3/2}\cdot dx_1. \quad (123)$$

The cable sections A and A_1 are here considered to be uniform in the respective spans. Usually $A = A_1$. Expanding the binomials and integrating, we obtain, with sufficient accuracy,

$$\int \frac{ds^3}{EA \cdot dx^2} = \frac{l'}{E_c A}(1+8n^2) + \frac{2l_2}{E_c A_1} \cdot \sec^3 \alpha_1 (1+8n_1^2), \quad (124)^*$$

where n and n_1 are the sag-ratios in main and side spans, respectively:

$$n = \frac{f}{l}, \quad n_1 = \frac{f_1}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (124')$$

Setting the values given by (122) and (124) in Eq. (121), and multiplying through by $\frac{3EI}{f^2 l}$, the formula for H becomes,

$$H = \frac{\frac{3}{f^2 l}\left[\int_0^l M' y dx + i \int_0^{l_1} M_1' y_1 dx_1\right]}{\frac{8}{5}(1+2irv^2) + \frac{3I}{Af^2}\frac{E}{E_c}\frac{l'}{l} \cdot (1+8n^2) + \frac{6I \cdot E \cdot l_2}{A_1 f^2 \cdot E_c \cdot l} \cdot \sec^3 \alpha_1 (1+8n_1^2)}. \quad (125)^*$$

where, for abbreviation,

$$i = \frac{I}{I_1}, \quad r = \frac{l_1}{l}, \quad v = \frac{f_1}{f}. \quad . \quad . \quad . \quad . \quad . \quad (126)$$

The elastic coefficient $E_c = E$, unless the cable is made of wire ropes. The denominator of Eq. (125), to be used for all suspension bridges of Type $2S$, will henceforth be designated by N. It is a constant for any given structure.

The second term in the numerator represents the contribution of any loads in the side spans, and will vanish if the side spans are built independent of the backstays. In the latter case

---

* If the cable section is not constant but varies with the cable stress (as in eyebar chains), change $8n^2$ to $\frac{16}{3}n^2$, $8n_1^2$ to $\frac{16}{3}n_1^2$, and $\sec^3 \alpha_1$ to $\sec^2 \alpha_1$; using A_0 (cable section at crown) instead of A and A_1.

the backstays will be straight (Type $2F$, Fig. 16), all terms containing y_1, f_1, n_1, or v will vanish, and Eq. (125) reduces to

$$H=\frac{\frac{3}{f^2l}\int_0^l M'ydx}{\frac{8}{5}+\frac{E}{E_c}\cdot\frac{3Il'}{Af^2l}(1+8n^2)+\frac{6I}{A_1f^2}\cdot\frac{E}{E_c}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1}. \quad . \quad (127)^*$$

where α_1 is the slope of the backstay.

19. Values of H for Special Cases of Loading.—In the preceding equations, the value of M' depends upon the loading in the particular case. Expressing M' as a function of x, using the value of y given by Eq. (14), and performing the integration as indicated, we find, for a single load P at a distance kl from either end of the span,

$$\int M'ydx=\tfrac{1}{3}Pfl^2k(1-2k^2+k^3). \quad . \quad . \quad . \quad (128)$$

Hence, by Eq. (125), for a concentration in the main span, the value of the horizontal tension will be,

$$H=\frac{1}{N\cdot n}\cdot B(k)\cdot P, \quad . \quad . \quad . \quad . \quad (129)$$

where N denotes the denominator of Eq. (125), and the function

$$B(k)=k(1-2k^2+k^3), \quad . \quad . \quad . \quad (129')$$

and may be obtained directly from Table I or from the graph in Fig. 12. The above value of H is a maximum when the load P is at the middle of the span; then $k=\frac{1}{2}$, and Eq. (129) yields,

$$\text{Max. } H=\frac{5}{16}\cdot\frac{1}{N\cdot n}\cdot P. \quad . \quad . \quad . \quad . \quad . \quad (130)$$

Similarly, for a concentration P in either side span, at a distance k_1l_1 from either end,

$$H=\frac{1}{N\cdot n}\cdot ir^2v\cdot B(k_1)\cdot P, \quad . \quad . \quad . \quad . \quad (131)$$

---

* If the cable section is not constant but varies with the cable stress (as in eyebar chains), change $8n^2$ to $\frac{16}{3}n^2$, and $\sec^3\alpha_1$ to $\sec^2\alpha_1$; using A_0 (cable section at crown) instead of A and A_1.

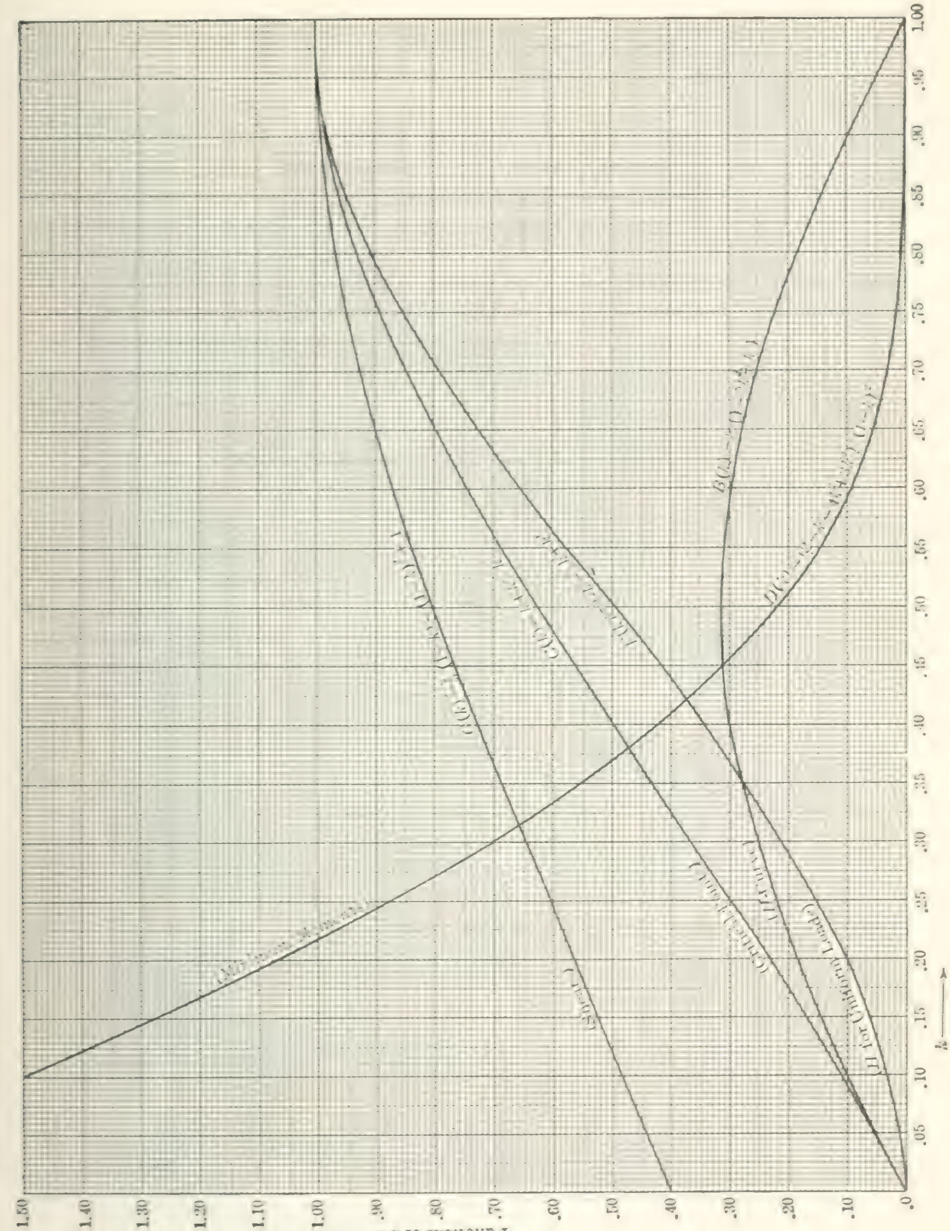


FIG. 12.—Graphs for the Solution of Suspension Bridge Formulas.

(See Table I, Page 42).

where $B(k_1)$ is the same function as defined by Eq. (129′). This value of H is a maximum when the load P is at the middle of the side span; then $k_1 = \frac{1}{2}$, and Eq. (131) yields,

$$\text{Max. } H = \frac{5}{16} \cdot \frac{1}{N \cdot n} \cdot ir^2 v \cdot P. \quad . \quad . \quad . \quad (132)$$

By plotting Eqs. (129) and (131) for different values of k and k_1, we obtain the H-curves or influence lines for H (Figs. 11, 14). The maximum ordinates of these curves are given by Eqs. (130) and (132).

For a uniform load of p pounds per foot, extending a distance kl from either end of the main span, we find, by integrating the function $B(k)$ in Eq. (129′),

$$H = \frac{1}{5N \cdot n} \cdot F(k) \cdot pl, \quad . \quad . \quad . \quad . \quad (133)$$

where the function,

$$F(k) = \tfrac{5}{2}k^2 - \tfrac{5}{2}k^4 + k^5, \quad . \quad . \quad . \quad . \quad . \quad (133')$$

and may be obtained directly from Table I or from the graph in Fig. 12. For $k = 1$, $F(k) = 1$.

For similar conditions in either side span, we find for a loaded length $k_1 l_1$,

$$H = \frac{1}{5Nn} \cdot ir^3 v \cdot F(k_1) \cdot p_1 l, \quad . \quad . \quad . \quad (134)$$

where $F(k_1)$ is the same function as defined by Eq. (133′).

The horizontal component of the cable tension will be a maximum when all spans are fully loaded, or when $k = 1$ and $k_1 = 1$. Hence, by Eqs. (133) and (134),

$$\text{Total } H = \frac{1}{5N \cdot n}(1 + 2ir^3 v)pl. \quad . \quad . \quad . \quad (135)$$

For a live load covering the central portion, JK, of the main span, from any section $x = jl$ to any other section $x = kl$, the application of Eq. (133) yields,

$$H = \frac{1}{5N \cdot n}[F(k) - F(j)] \cdot pl, \quad . \quad . \quad . \quad (136)$$

where $F(j)$ and $F(k)$ are the same function as defined by Eq. (133′).

The graph of $F(k)$ in Fig. 12 shows the proportional increase in the value of H as a uniform load comes on and fills the main span (or either side span). The difference between the two ordinates for any sections, J and K, multiplied by $\frac{pl}{5Nn}\left(\text{or by } \frac{p_1lir^3v}{5Nn}\right)$, will give the value of H for the corresponding partial loading JK.

For opposite loading conditions, that is, load covering both side spans and all of the main span with the exception of the central portion JK, we find the value of H by subtracting the members of Eq. (136) from those of Eq. (135):

$$H = \frac{pl}{5N \cdot n}[1 - F(k) + F(j)] + \frac{2p_1l}{5Nn} \cdot ir^3v. \quad . \quad . \quad (137)$$

20. Moments in the Stiffening Truss.—The bending moment at any section (main or side span) is given by Eq. (82),

$$M = M' - Hy, \quad M_1 = M_1' - Hy_1. \quad . \quad . \quad . \quad (138)$$

If any span is free from load, the moments for that span are obtained by placing M' (or M_1') equal to zero, giving,

$$M = -Hy, \quad M_1 = -Hy_1. \quad . \quad . \quad . \quad . \quad (139)$$

where H is the cable tension produced by loads in the other spans, or by temperature.

With all three spans loaded, using the value of H given by Eq. (135), Eq. (138) yields, for any section in the main span,

$$\text{Total } M = \tfrac{1}{2}px(l-x)\left[1 - \frac{8}{5N}(1 + 2ir^3v)\right]. \quad . \quad . \quad (140)$$

and, for any section in the side span,

$$\text{Total } M_1 = \tfrac{1}{2}px_1(l_1 - x_1)\left[1 - \frac{8}{5N}(1 + 2ir^3v)\frac{v}{r^2}\right] \quad . \quad (141)$$

The influence diagrams for bending moment are constructed, in accordance with Eq. (86), by superimposing the influence triangle for $\frac{M'}{y}$ on the H-influence curve: The H-curve is

TABLE I

FUNCTIONS OCCURRING IN SUSPENSION BRIDGE FORMULAS

	H Influence Line	Critical Points	Minimum Moments	H for Uniform Loads	Shears	
k	$B(k)$	$C(k)$	$D(k)$	$F(k)$	$G(k)$	k
	$k(1-2k^2+k^3)$	$k+k^2-k^3$	$(2-k-4k^2+3k^3)(1-k)^2$	$\frac{5}{2}k^2-\frac{5}{2}k^4+k^5$	$\frac{2}{5}(1-k)^3-(1-k)^2+1$	
0	0	0	2.0	0	0.4	0
.05	.0498	.0524	1.7511	.0062	.4404	.05
.1	.0981	.1090	1.5090	.0248	.4816	.10
.15	.1438	.1691	1.2790	.0550	.5232	.15
.2	.1856	.2320	1.0650	.0963	.5648	.20
.25	.2227	.2969	.8704	.1474	.6062	.25
.3	.2541	.3630	.6962	.2072	.6472	.30
.35	.2793	.4296	.5445	.2740	.6874	.35
.4	.2976	.4960	.4147	.3462	.7264	.40
.45	.3088	.5614	.3065	.4222	.7640	.45
.5	.3125	.6250	.2188	0.5	.8000	.50
.55	.3088	.6861	.1497	.5778	.8340	.55
.6	.2976	.7440	.0973	.6538	.8656	.60
.65	.2793	.7979	.0593	.7260	.8946	.65
.7	.2541	.8470	.0332	.7928	.9208	.70
.75	.2227	.8906	.0166	.8526	.9438	.75
.8	.1856	.9280	.0070	.9037	.9632	.80
.85	.1438	.9584	.0023	.9450	.9788	.85
.9	.0981	.9810	.0005	.9752	.9904	.90
.95	.0498	.9951	.0003	.9938	.9976	.95
1.00	0	1.0	0	1.0	1.0	1.0

plotted with ordinates given by Eqs. (129) and (131); the $\frac{M'}{y}$ triangles have a constant height, $\frac{l}{4f}$ in the main span and $\frac{l_1}{4f_1}$ in the side spans. The resulting influence diagrams are shown in Figs. 11*b* and 11*c*. The intercepted areas, multiplied by py, give the desired bending moments; areas below the H-curve represent positive or maximum moments, and those above represent negative or minimum moments.

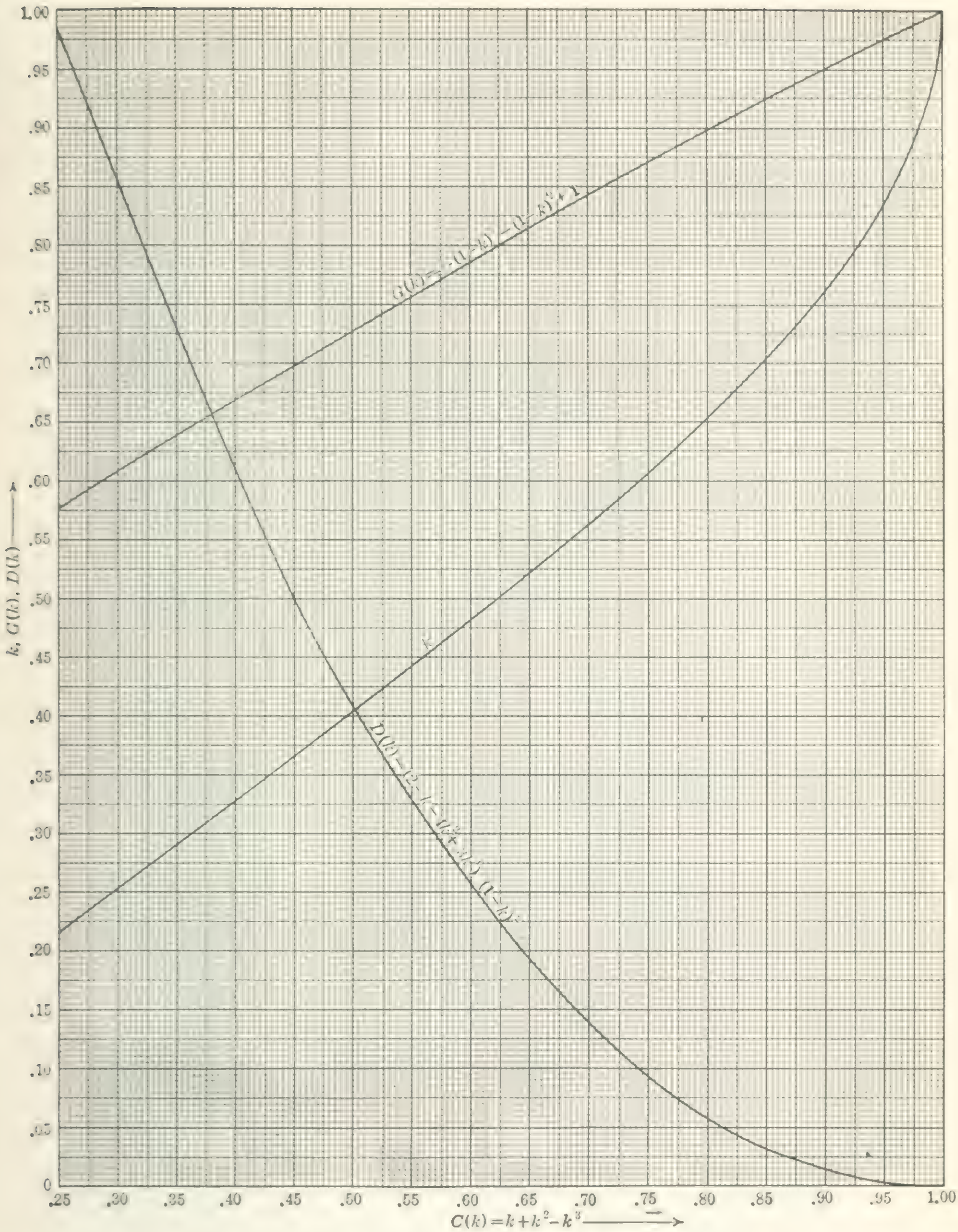


FIG. 13.—Graphs for the Solution of Suspension Bridge Formulas.
(Supplementary to Fig. 12).

For any section in the main span, there is a zero point or critical point K (Fig. 11b), represented by the intersection of the superimposed influence lines. The distance kl to this critical point is given by the relation,

$$C(k)=k+k^2-k^3=N\cdot n\cdot\frac{x}{y}. \quad . \quad . \quad . \quad (142)$$

Values of the function $C(k)$ are listed in Table 1 and plotted in a graph in Figs. 12 and 13, to facilitate the solution of Eq. (142) for k.

The maximum negative moment at any section of the main span is obtained by loading the length $l-kl$ in that span and completely loading both side spans (Fig. 11b). Then, using the values of Eqs. (133) and (135), Eq. (138) yields,

$$\text{Min. } M=-\frac{2px(l-x)}{5N}[D(k)+4ir^3v], \quad . \quad . \quad (143)$$

where the function,

$$D(k)=(2-k-4k^2+3k^3)(1-k)^2, \quad . \quad . \quad (143')$$

and is given, for different values of k, by Table I and by the graph in Fig. 12 or 13. The value of k or $C(k)$ obtained from Eq. (142) is to be used.

Equation (143) applies to all sections from $x=0$ to $x'=\frac{N}{4}\cdot l$. For the minimum moments at the sections near the center, from x' to $l-x'$, it is necessary to bring on some load also from the left end of the span, as there are two critical points, K' and K'', for these sections (see dotted diagram, Fig. 11b); so that the expression (143) for these moments must be corrected by replacing $D(k)$ by $D(k')+D(k'')$, where k' is the value of k (Eq. 142) corresponding to the given section x, and k'' is the value of k corresponding to the symmetrically located section $(l-x)$.

The maximum positive moments are given by the relation,

$$\text{Max. } M=\text{Total } M-\text{Min. } M. \quad . \quad . \quad . \quad . \quad (144)$$

Subtracting the values given by Eq. (143) from those given by Eq. (140), we obtain,

$$\text{Max. } M = \tfrac{1}{2}px(l-x)\left[1 - \frac{8}{5N}[1 - \tfrac{1}{2}D(k)]\right]. \quad . \quad (144')$$

The loading corresponding to this moment is indicated in Fig. 11*b*; only a portion of the main span is loaded, the side spans being without load.

There are no critical points in the side spans. For the greatest negative moment at any section x_1 in one of the side spans, load the other two spans (Fig. 11*c*), giving,

$$\text{Min. } M_1 = -y_1 \cdot \frac{1 + ir^3v}{5Nn} \cdot pl. \quad . \quad . \quad . \quad . \quad . \quad (145)$$

Loading the span itself produces the greatest positive moments, which are obtained by the relation,

$$\text{Max. } M_1 = \text{Total } M_1 - \text{Min. } M_1. \quad . \quad . \quad . \quad . \quad . \quad (146)$$

Subtracting the values given by Eq. (145) from those given by Eq. (141), we obtain,

$$\text{Max. } M_1 = \frac{ry_1}{8n_1}\left(1 - \frac{8}{5N}irv^2\right) \cdot pl. \quad . \quad . \quad . \quad . \quad . \quad (146')$$

The maximum and minimum moments for the various sections of a stiffening truss (Type 2*S*), as calculated from Eqs. (143), (144), (145) and (146), are plotted in Fig. 11*d*, to serve as a guide in proportioning the chord members.

21. Shears in the Stiffening Truss.—With the three spans completely loaded, the shear at any section x in the main span will be, by Eqs. (84), (90) and (135),

$$\text{Total } V = \tfrac{1}{2}p(l-2x)\left[1 - \frac{8}{5N}(1 + 2ir^3v)\right]. \quad . \quad . \quad . \quad (147)$$

and, in the side spans,

$$\text{Total } V_1 = \tfrac{1}{2}p(l_1 - 2x_1)\left[1 - \frac{8}{5N} \cdot \frac{v}{r^2} \cdot (1 + 2ir^3v)\right]. \quad (148)$$

The influence diagram for shear at any section is constructed according to Eq. (89), by superimposing on the H-curve (Eqs.

129 and 131) the influence lines for $\frac{V}{\tan\phi}$. The latter will have end intercepts $= \cot\phi$, where ϕ is the slope of the cable at the given section. The resulting influence diagram is shown in Fig. 14*a*. The intercepted areas, multiplied by $p \cdot \tan\phi$, give the desired vertical shears V. Areas below the H-curve repre-

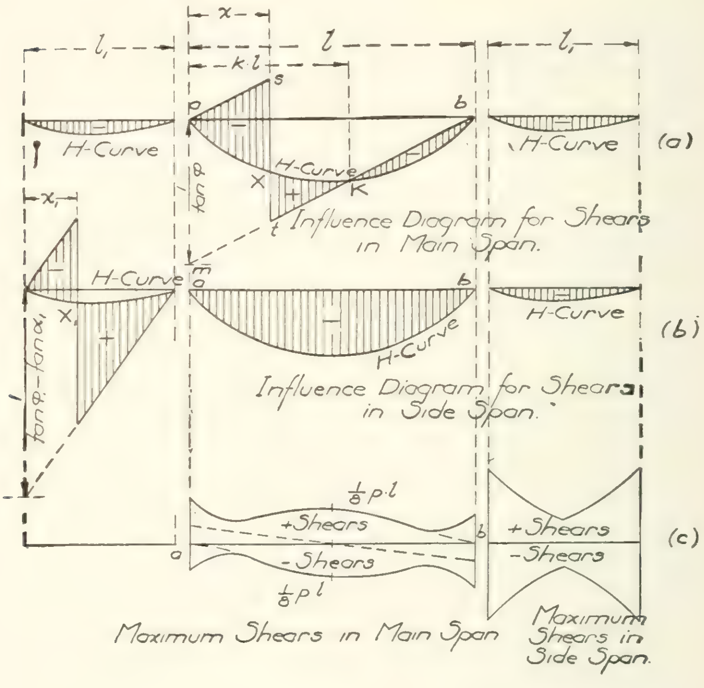


FIG. 14.—Shear Diagrams for Two-hinged Stiffening Truss. (Type 2*S*).

sent positive or maximum shears, and areas above represent negative or minimum shears.

Loading the main span from the given section X to the end of the span, we obtain the maximum positive shears by Eqs. (84), (90) and (133),

$$\text{Max. } V = \tfrac{1}{2}pl\left(1-\frac{x}{l}\right)^2\left[1-\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G\left(\frac{x}{l}\right)\right], \quad . \quad (149)$$

where the function,

$$G\left(\frac{x}{l}\right)=\frac{2}{5}\left(1-\frac{x}{l}\right)^3-\left(1-\frac{x}{l}\right)^2+1, \quad . \quad . \quad . \quad . \quad . \qquad (149')$$

and is given by Table I and the graph in Fig. 12.

For the sections near the ends of the span, from $x=0$ to $x=\frac{l}{2}\left(1-\frac{N}{4}\right)$, the loads must not extend to the end of the span to produce the maximum positive shears, but must extend only to a point K (Fig. 14a) whose abscissa $x=kl$ is determined by the following equation:

$$C(k)=k+k^2-k^3=\frac{N}{4}\cdot\frac{l}{l-2x}. \quad . \quad . \quad . \quad . \qquad (150)$$

For these sections, the positive shears given by Eq. (149) must be increased by an amount,

$$\text{Add. } V=\tfrac{1}{2}pl(1-k)^2\cdot\left[\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G(k)-1\right]. \quad . \quad . \qquad (151)$$

where the function,

$$G(k)=\tfrac{2}{5}(1-k)^3-(1-k)^2+1, \quad . \quad . \quad . \quad . \quad . \qquad (151')$$

and, like the same function in Eq. (149'), is given by Table I and the graph in Fig. 12 or 13.

Formula (150) for the critical section is solved in the same manner as Eq. (142) with the aid of Table I or the graph in Fig. 12 or 13.

There are no critical points for shear in the side spans. The influence diagram (Fig. 14b) shows the conditions of loading. For maximum shear at any section x_1, the load extends from the section to the tower, giving,

$$\text{Max. } V_1=\tfrac{1}{2}pl_1\left(1-\frac{x_1}{l_1}\right)^2\left[1-\frac{8}{N}\,ir^2\left(\frac{1}{2}-\frac{x_1}{l_1}\right)\cdot G\left(\frac{x_1}{l_1}\right)\right], \qquad (152)$$

where $G\left(\frac{x_1}{l_1}\right)$ is the same function as defined by Eqs. (149') and (151').

The maximum negative shears in main and side spans are given by the relations,

$$\text{Min. } V = \text{Total } V - \text{Max. } V, \quad . \quad . \quad . \quad (153)$$

and

$$\text{Min. } V_1 = \text{Total } V_1 - \text{Max. } V_1. \quad . \quad . \quad (153')$$

The maximum positive and negative shears for different sections of the main and side spans, as given by Eqs. (149), (152), (153) and (153′), are plotted for a typical suspension bridge, in Fig. 14*c*, to serve as a guide in proportioning the web members.

22. Temperature Stresses.—The total length of cable between anchorages is, by Eqs. (22) and (26),

$$L = l(1 + \tfrac{8}{3}n^2) + 2l_1\left(\sec\alpha_1 + \frac{8}{3}\cdot\frac{n_1^2}{\sec^3\alpha_1}\right), \quad . \quad . \quad (154)$$

Corrections should be made in the value of L for any portions of the cable not included in the spans l or l_1.

Under the influence of a rise in temperature, the total increase in length between anchorages will be:

$$\Delta = \omega t L. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

Substituting this value for the numerator in Eqs. (116) to (125), we obtain,

$$H_t = -\frac{3EI\cdot\omega t L}{f^2\cdot N\cdot l}, \quad . \quad . \quad . \quad . \quad . \quad (156)$$

where N denotes the denominator of Eq. (125) and L is given by Eq. (154). (For an extreme variation of $t = \pm 60°$ F., $E\omega t = 11{,}720$.)

The resulting bending moment at any section of the truss is given by,

$$M_t = -H_t\cdot y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$

and the vertical shear by,

$$V_t = -H_t(\tan\phi - \tan\alpha), \quad . \quad . \quad (158)$$

where ϕ is the inclination of the cable at the given section, and α is the inclination of the cable chord (Eqs. 84′, 90).

23. Deflections of the Stiffening Truss.—For any specified loading, the deflections of the stiffening truss may be computed

as the difference between the downward deflections produced by the applied loads and the upward deflections produced by the suspender forces, the stiffening truss being treated as a simple beam (for Types $2F$ and $2S$). The suspender forces are equivalent to an upward-acting load, uniformly distributed over the entire span, and, by Eq. (78), amounting to,

$$s=\frac{8f}{l^2}\cdot H. \qquad (78)$$

For a uniform load p covering the main span, the resultant effective load acting on the stiffening truss will be, by Eqs. (78) and (135),

$$p-s=p\left(1-\frac{8}{5N}\right). \qquad (159)$$

and the resulting deflection will be,

$$d=\frac{5}{384}\left(1-\frac{8}{5N}\right)\frac{pl^4}{EI}. \qquad (160)$$

In the general case, the applied loads will produce a deflection at a distance x of,

$$d'=\frac{l-x}{l}\int_0^x\frac{M'}{EI}\,xdx+\frac{x}{l}\int_x^l\frac{M'}{EI}(l-x)\cdot dx. \qquad (161)$$

The suspender forces, given by Eq. (78), will produce an upward deflection, at a distance x, of,

$$d''=\frac{1}{3EI}x(l^3-2lx^2+x^3)\frac{f}{l^2}\cdot H. \qquad (162)$$

It should be noted that this deflection curve (Eq. 162) is similar to the H-influence curve given by Eq. (129). Using the function defined by Eq. (129′), Eq. (162) may be written,

$$d''=\frac{fl^2}{3EI}\cdot B\left(\frac{x}{l}\right)\cdot H. \qquad (162')$$

The resulting deflection of the truss at any point will then be obtained from Eqs. (161) and (162) as

$$d=d'-d''. \qquad (163)$$

Equation (160), for a full-span load, may be derived directly from Eq. (163).

If merely the half-span is loaded with p per unit length, then the deflection at the quarter-point will be, by Eqs. (161) and (162), in the loaded half,

$$d=\frac{1}{6144}\left(31-\frac{57}{2}\cdot\frac{8}{5N}\right)\cdot\frac{pl^4}{EI}, \quad . \quad . \quad . \quad . \qquad (164)$$

and, in the unloaded half,

$$d=-\frac{1}{6144}\left(\frac{57}{2}\cdot\frac{8}{5N}-26\right)\frac{pl^4}{EI}. \quad . \quad . \qquad (164')$$

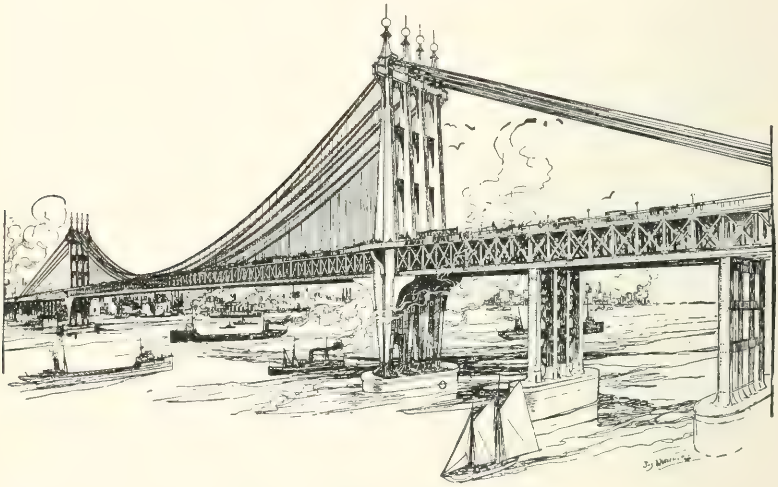


FIG. 15.—Detroit–Windsor Bridge.
(Type $2F$).

Span 1803 feet. Combined Railway and Highway Bridge. 8 Cables. C. E. Fowler, Chief Engineer. D. B. Steinman, Associate Engineer. W. H. Burr, G. H. Pegram, C. N. Monsarrat and C. R. Young, Consulting Engineers.

By Eq. (125), N will always be greater than $\frac{8}{5}$. Substituting this minimum value in Eq. (164) or (164′), we obtain the upward or downward deflections at the quarter-points:

$$d=\frac{1}{2}\cdot\frac{5}{384}\cdot\frac{p}{EI}\cdot\left(\frac{l}{2}\right)^4. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (165)$$

The deflections produced by temperature effects, or by a yielding of the anchorages, are given by Eq. (162′), upon substituting for H the horizontal tension caused by the given influence. Substituting the expression from Eq. (156), we obtain,

$$d''=B\left(\frac{x}{l}\right)\cdot\frac{\Delta L}{N\cdot n}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (166)$$

where the function $B\left(\frac{x}{l}\right)$ is defined by Eq. (129′) and is given by Table I and the graph in Fig. 12.

24. Straight Backstays (Type 2F).—If the stiffening truss is built independent of the cables in the side spans (Figs. 15, 16),

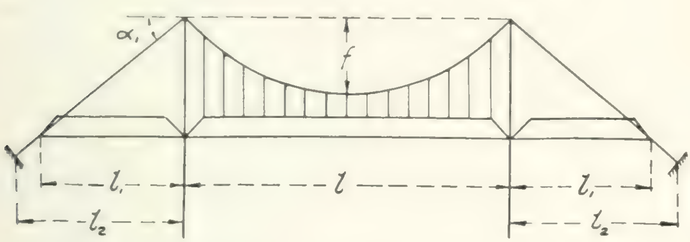


FIG. 16.—Two-hinged Stiffening Truss with Straight Backstays. (Type 2F).

the backstays will be straight and $f_1=0$. Consequently all terms containing f_1, y_1, $n_1=\frac{f_1}{l_1}$, or $v=\frac{f_1}{f}$ will vanish in Eqs. (125) to (166) inclusive.

The side spans will then act as simple beams, unaffected by any loads in the other spans; and the main-span and cable stresses will be unaffected by any loads in the side spans.

The denominator of the general expression for H (Eq. 125) will then reduce to the denominator of Eq. (127):

$$N=\frac{8}{5}+\frac{3I}{Af^2}\frac{E}{E_c}\frac{l'}{l}(1+8n^2)+\frac{6I}{A_1f^2}\frac{E}{E_c}\frac{l_2}{l}\cdot\sec^3\alpha_1. \qquad (167)^*$$

Equations (131), (134), and (145), will vanish.

* See Footnote to Eq. 127.

The maximum value of H will be produced by a uniform load p covering the main span, and will be, by Eq. (135),

$$\text{Total } H = \frac{pl}{5N \cdot n}. \qquad (168)$$

The bending moment at any section x of the main span will then be, by Eq. (140),

$$\text{Total } M = \tfrac{1}{2}px(l-x)\left(1 - \frac{8}{5N}\right). \qquad (169)$$

The greatest negative bending moment will be, by Eq. (143),

$$\text{Min. } M = -\frac{2px(l-x)}{5N} \cdot D(k). \qquad (170)$$

The greatest positive moment is then given by Eq. (144′), or by,

$$\text{Max. } M = \text{Total } M - \text{Min. } M. \qquad (171)$$

In the side spans, there will be no negative moments. The greatest positive moments will be, by Eq. (141),

$$\text{Max. } M_1 = \text{Total } M_1 = \tfrac{1}{2}p_1x_1(l_1 - x_1), \qquad (172)$$

exactly as in a simple beam.

With load covering the entire span, the shears in the main span will be, by Eq. (147),

$$\text{Total } V = \tfrac{1}{2}p(l - 2x)\left(1 - \frac{8}{5N}\right), \qquad (173)$$

and, in the side spans, by Eq. (148),

$$\text{Total } V_1 = \tfrac{1}{2}p_1(l_1 - 2x_1). \qquad (174)$$

The maximum shears in the main span will be given by Eqs. (149), (150) and (151). In the side spans, the maximum shears will be, by Eq. (152),

$$\text{Max. } V_1 = \tfrac{1}{2}p_1l_1\left(1 - \frac{x_1}{l_1}\right)^2, \qquad (175)$$

exactly as in a simple beam.

The total length of cable will be, by Eq. (154),

$$L = l'(1 + \tfrac{8}{3}n^2) + 2l_2 \cdot \sec \alpha_1, \qquad (176)$$

and the temperature stresses are then given by Eqs. (156), (157) and (158).

SECTION VI. HINGELESS STIFFENING TRUSSES

(Types $0F$ and $0S$)

25. Fundamental Relations. Hingeless stiffening trusses (Figs. 17, 18) are continuous at the towers; hence there will be bending moments in the truss at the towers.

The moments and shears at any section in the stiffening truss will be the resultants of the values produced by the downward-acting loads (M' and V') and the upward-acting suspender

FIG. 17.—Suspension Bridge over the Rhine at Cologne.
(Type $0S$).

Continuous Stiffening Girder. Eyebar Chains. Self-anchored. Rocker Towers.
Span 605 feet. Erected 1915.

forces (M_s and V_s). Equations (78), (79) and (80) will apply; but the continuity of the truss must be taken into account in calculating the respective moments and shears.

If M_1 and M_2 are the bending moments at the towers produced by the downward loads on the stiffening truss, and if M_0 is the simple-beam bending moment at any section x, then the

resultant bending moment due to the downward loads acting on the continuous truss will be,

$$M' = M_0 + \frac{l-x}{l} M_1 + \frac{x}{l} \cdot M_2, \quad . \quad . \quad . \quad (177)$$

in the main span, and,

$$M' = M_0 + \frac{x_1}{l_1} M_{1,2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (177')$$

in the side spans.

The upward-acting suspender forces will be uniform over each span. For any value of H, by Eq. (78), the upward pull will be $H \cdot \frac{8f}{l^2}$ per lineal foot in the main span and $H \cdot \frac{8f_1}{l_1^2}$ in the side spans. Then, by the Theorem of Three Moments for uniform load conditions, we find the moments at the towers (for symmetrical spans) to be,

$$-H \cdot m_1 = -H \cdot m_2 = -H \cdot (e \cdot f), \quad . \quad . \quad . \quad (178)$$

in which the coefficient of f is a constant defined by,

$$e = \frac{2 + 2irv}{3 + 2ir}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (179)$$

where i, r, and v are defined by Eq. (126).

The simple-beam bending moment produced by the suspender forces is given by Eq. (81) as $H \cdot y$. Adding the correction for the end moments at the towers (Eq. 178), we obtain the resultant suspender moments as,

$$M_s = H \cdot (y - e \cdot f), \quad . \quad . \quad . \quad . \quad . \quad . \quad (180)$$

for any section in the main span; and, for any section in the side-spans,

$$M_s = H \cdot \left(y_1 - \frac{x_1}{l_1} \cdot ef \right), \quad . \quad . \quad . \quad . \quad (181)$$

where x_1 is measured from the free end of the span, and y_1 is the vertical ordinate of the side cable below the connecting chord $D'A'$ (Fig. 18*a*).

Substituting (177), (177'), (180) and (181) in Eq. (79), we have, for bending moments in the main span, (Fig. 19),

$$M = M_0 + \frac{l-x}{l} M_1 + \frac{x}{l} \cdot M_2 - H(y - cf), \quad . \quad . \quad . \quad (182)$$

and, for bending moments in the side span,

$$M = M_0 + \frac{x_1}{l_1} \cdot M_{1,2} - H\left(y_1 - \frac{x_1}{l_1} \cdot cf\right). \quad . \quad . \quad . \quad (183)$$

If any span is without load, M_0 for that span will vanish.

The shears produced by the downward-acting loads will be,

$$V' = V_0 + \frac{M_2 - M_1}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (184)$$

in the main span, and

$$V' = V_0 + \frac{M_1}{l_1}, \quad \text{or} \quad V' = V_0 - \frac{M_2}{l_1}, \quad . \quad . \quad . \quad (185)$$

in the side spans. In these equations, V_0 denotes the simple-beam shears for the given loading.

The shears produced by the upward-acting suspender forces will be

$$V_s = H(\tan\phi - \tan\alpha) \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

in the main span, and

$$V_s = H\left(\tan\phi_1 - \tan\alpha_1 - \frac{cf}{l_1}\right) \quad . \quad . \quad . \quad . \quad (187)$$

in the side spans.

Substituting (184), (185), (186) and (187) in Eq. (80), we have, for resultant shears in the main span,

$$V = V_0 + \frac{M_2 - M_1}{l} - H \cdot (\tan\phi - \tan\alpha), \quad . \quad . \quad . \quad (188)$$

and, for resultant shears in the side spans,

$$V = V_0 \pm \frac{M_{1,2}}{l_1} - H \cdot \left(\tan\phi_1 - \tan\alpha_1 - \frac{cf}{l_1}\right). \quad . \quad (189)$$

If any span is without load, V_0 for that span will vanish. If the two towers are of equal height, then, in the main span, $\alpha = 0$.

26. Moments at the Towers (Types 0*F* and 0*S*).—The values of the end-moments M_1 and M_2, used in Eqs. (177) to (189), may be determined, for any given loading, by the Theorem of Three Moments.

For a concentration P in the main span, at a distance kl from the left tower, we thus obtain,

$$M_1 = -Pl\cdot k(1-k)\frac{(3+2ir)(1-k)+2ir}{(3+2ir)(1+2ir)}, \quad . \quad (190)$$

$$M_2 = -Pl\cdot k(1-k)\frac{(3+2ir)k+2ir}{(3+2ir)(1+2ir)}. \quad . \quad . \quad (191)$$

The sum of these two end-moments will be,

$$M_1+M_2 = -\frac{3Pl\cdot k(1-k)}{3+2ir}, \quad . \quad . \quad . \quad . \quad (192)$$

and the difference will be,

$$M_1-M_2 = -Plk(1-k)\frac{1-2k}{1+2ir}. \quad . \quad . \quad . \quad (193)$$

For a concentration P in the left side span, at a distance kl_1 from the outer end, the Theorem of Three Moments yields:

$$M_1 = -Pl\frac{2ir^2(1+ir)(k-k^3)}{(3+2ir)(1+2ir)}, \quad . \quad . \quad . \quad (194)$$

$$M_2 = +Pl\frac{ir^2(k-k^3)}{(3+2ir)(1+2ir)}. \quad . \quad . \quad . \quad (195)$$

For a uniform load covering the main span, we obtain,

$$M_1 = M_2 = -\frac{pl^2}{4(3+2ir)}. \quad . \quad . \quad . \quad . \quad (196)$$

For a uniform load covering the left side span, we obtain,

$$M_1 = -\frac{p_1l^2}{4}\cdot\frac{2ir^3(1+ir)}{(3+2ir)(1+2ir)}, \quad . \quad . \quad . \quad (197)$$

$$M_2 = +\frac{p_1l^2}{4}\cdot\frac{ir^3}{(3+2ir)(1+2ir)}. \quad . \quad . \quad . \quad (198)$$

For a uniform load covering all three spans, we obtain,

$$M_1 = M_2 = -\frac{pl^2}{4}\cdot\frac{1+ir^3}{3+2ir}. \quad . \quad . \quad . \quad . \quad (199)$$

27. The Horizontal Tension H.—The general formula (117) for the horizontal tension H is applicable to the continuous stiffening truss (Types OF and OS).

Equation (118), for bending moments produced by the suspender forces, is now replaced by the expressions (180) and (181), and Eq. (119) becomes,

$$m = -y + cf, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

for the main span, and

$$m = -y_1 + \frac{x_1}{l_1} \cdot cf, \quad . \quad . \quad . \quad . \quad . \quad . \quad (201)$$

for the side spans.

Substituting these values and integrating over all three spans, we obtain, as a substitute for Eq. (122),

$$\int \frac{m^2}{EI} dx = \frac{f^2 l}{3EI}\left(\tfrac{8}{5} - 4c + 3c^2\right) + \frac{2f^2 l_1}{3EI_1}\left(\tfrac{8}{5}v^2 - 2cv + c^2\right). \quad (202)$$

Equations (120), (123), (124) and (124′) are retained unchanged. Collecting all the values and substituting in Eq. (117), we obtain the expression for H in the continuous type of suspension bridge (in place of Eq. 125):

$$H = \frac{\frac{3}{f^2 l}\left[\int_0^l M'(y - cf)dx + i\int_0^{l_1} M_1'\left(y_1 - \frac{x_1}{l_1}cf\right)dx_1\right]}{\tfrac{8}{5} - 4c + 3c^2 + 2ir\left(\tfrac{8}{5}v^2 + c^2 - 2cv\right) + \frac{E}{E_c}\frac{3I'}{Af^2 l}(1 + 8n^2) + \frac{E}{E_c}\frac{6I}{A_1 f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1(1 + 8n_1^2)}. \quad . \quad (203)^*$$

The denominator of this expression is a constant for a given structure, and will henceforth be denoted by N. (If hinges are inserted at the towers, the coefficient of continuity c will be zero, and Eq. [203] reduces to Eq. [125]).

28. Values of H for Special Cases of Loading. In the last equation (203), the value of the numerator depends upon the loading in any particular case. Expressing M' as a function of x (Eq. 177), substituting the value of y given by Eq. (14), and

* See Footnote to Eq. (125).

performing the integration as indicated, we find, for a single load P at a distance kl from either end of the main span,

$$H = \frac{1}{Nn}[B(k) - \tfrac{3}{2}e(k-k^2)]\cdot P, \qquad (204)$$

where N denotes the denominator of Eq. (203), and the function $B(k)$ is defined by Eq. (129′) and is given by Table I and Fig. 12.

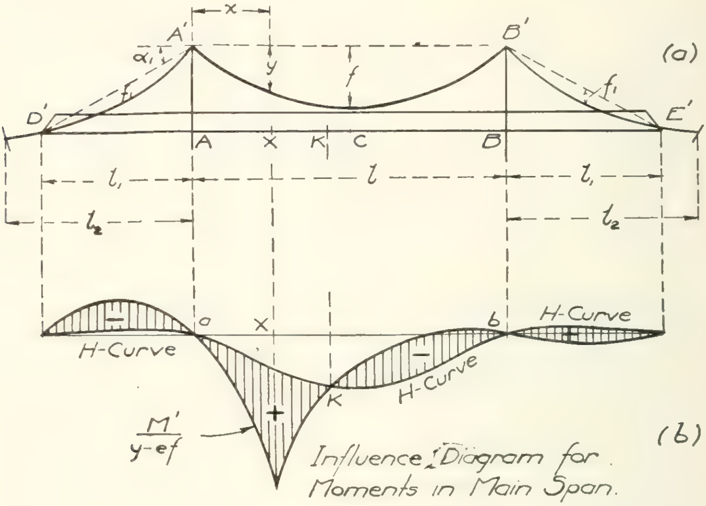


FIG. 18.—Moment Diagram for Continuous Stiffening Truss. (Type $0S$).

Similarly, for a concentration P_1 in either side span, at a distance k_1l_1 from the free end, we obtain,

$$H = \frac{ir^2}{Nn}\left[v\cdot B(k_1) - \frac{e}{2}(k_1 - k_1{}^3)\right]\cdot P_1. \qquad (205)$$

Plotting Eqs. (204) and (205), we obtain the H-influence line, Fig. 18b.

If the main span is completely loaded, we obtain, by integrating Eq. (204),

$$H = \frac{1}{N\cdot n}\left(\frac{1}{5} - \frac{e}{4}\right)\cdot pl. \qquad (206)$$

If both side spans are completely loaded, we obtain, by integrating Eq. (205),

$$H = \frac{2ir^3}{Nn}\left(\frac{1}{5} - \frac{e}{8}\right) \cdot p_1 l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (207)$$

If the main span is loaded for a distance kl from either tower, we obtain, from Eq. (204),

$$H = \frac{1}{5N \cdot n}\left[F(k) - \frac{5e}{4}(3 - 2k)k^2\right] \cdot pl, \quad . \quad . \quad (208)$$

where $F(k)$ is defined by Eq. (133′) and is given by Table I and Fig. 12.

If either side span is loaded for a distance $k_1 l_1$ from the free end, we obtain, from Eq. (205),

$$H = \frac{1}{5Nn} \cdot ir^3[2 \cdot F(k_1) - \tfrac{5}{4}e(2 - k_1^2) \cdot k_1^2]p_1 l, \quad . \quad (209)$$

where $F(k_1)$ is the same function as defined by Eq. (133′).

In the foregoing equations, N represents the denominator of Eq. (203).

(If the stiffening truss is interrupted at the towers, the factor of continuity $e = 0$, and the above formulas reduce to the corresponding equations [129] to [135] for the two-hinged stiffening truss.)

29. Moments in the Stiffening Truss.—With all three spans loaded, the bending moment at any section of the main span is given, very closely, by Eqs. (182) and (199), as,

$$\text{Total } M = \left(\tfrac{1}{2}p - H \cdot \frac{4f}{l^2}\right)x \cdot (l - x) - e(\tfrac{1}{8}pl^2 - H \cdot f), \quad (210)$$

and, at any section of the side span distant x_1 from the free end by Eqs. (183) and (199), as,

$$\text{Total } M = \left(\tfrac{1}{2}p - H \cdot \frac{4f_1}{l_1^2}\right)x_1(l_1 - x_1) - e(\tfrac{1}{8}pl^2 - Hf)\frac{x_1}{l_1}, \quad (211)$$

where e is defined by Eq. (179), and H is given by the combination of Eqs. (206) and (207).

The moments for other loadings must be calculated by the

general Eqs. (182) and (183), with the values of H given by Eqs. (204) to (209), and the values of M_1 and M_2 given by Eqs. (190) to (199).

Influence lines for moments may be drawn as in the previous cases. For moments in the main span, Eq. (182) is written in the form,

$$M = \left[\frac{M_0 + M_1 \frac{l-x}{l} + M_2 \cdot \frac{x}{l}}{y - ef} - H \right] \cdot (y - ef), \quad . \quad (212)$$

thus giving the bending moments as $(y-cf)$ times the intercepts obtained by superimposing the influence line for $\frac{M'}{y-ef}$ upon the influence line for H. This construction is indicated in Fig. 18b. For moments in the side spans, the corresponding influence line equation is obtained from Eq. (183):

$$M = \left[\frac{M_0 + \frac{x_1}{l_1} \cdot M_{1,2}}{y_1 - \frac{x_1}{l_1} \cdot cf} - H \right] \left(y_1 - \frac{x_1}{l_1} \cdot cf \right). \quad . \quad . \quad (213)$$

For the continuous stiffening truss, the influence line method just outlined is not very convenient, as the M' influence line (Fig. 18b) is a curve for which there is no simple, direct method of plotting.

A more convenient method is that of the Equilibrium Polygon constructed with pole-distance H, corresponding to Eq. (85) and Fig. 7. For the continuous stiffening truss, this construction is modified as follows (Fig. 19): At a distance cf below the closing chord $A'B'$, a base line AB is drawn, so that the cable ordinates measured from this base line will be $(y-cf)$ and will therefore represent M_s (Eq. 180). The equilibrium polygon $A''MB''$ for any given loads is then constructed upon the same base line, with the same pole-distance H; the height AA'' represents $-M_1$, the height BB'' represents $-M_2$, and the polygon ordinates below $A''B''$ represent M_0: hence, by Eq. (177), the ordinates measured below the base line AB represent M'. Then, by Eq. (79), the intercept between the cable curve and

the superimposed equilibrium polygon, *multiplied by H*, will give the resultant bending moment M at any section.

For a single concentrated load P, the equilibrium polygon $A''MB''$ is a triangle, and the M intercepts can easily be scaled or figured. By moving a unit load P to successive panel points, we thus obtain a set of influence values of M for all sections.

The corresponding construction in the side spans is also indicated in Fig. 19.

30. Temperature Stresses.—The horizontal tension produced by a rise in temperature of $t°$ is given by,

$$H_t = -\frac{3EI\omega tL}{f^2 \cdot N \cdot l}, \quad . \quad . \quad . \quad . \quad . \quad (214)$$

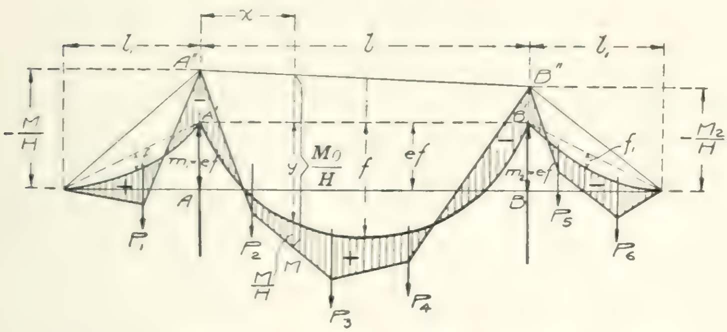


FIG. 19.—Equilibrium Polygon for Continuous Stiffening Truss. (Type OS).

where N is the denominator of Eq. (203), and L is given by Eq. (154).

The resulting moments in the stiffening truss will be given, by Eqs. (180) and (181), as,

$$M_t = -H_t \cdot (y - ef), \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

for the main span, and,

$$M_t = -H_t\left(y_1 - \frac{x_1}{l_1} \cdot ef\right), \quad . \quad . \quad . \quad . \quad . \quad (216)$$

for the side spans.

The vertical shears are given by Eqs. (186) and (187) as,

$$V_t = -H_t(\tan\phi - \tan\alpha), \quad . \quad . \quad . \quad . \quad . \quad (217)$$

for the main span, and

$$V_t = -H_t\left(\tan\phi_1 - \tan\alpha_1 - \frac{ef}{l_1}\right), \quad . \quad . \quad . \quad . \qquad (218)$$

for the side spans.

31. Straight Backstays (**Type** $0F$).—If the stiffening truss in the side spans is built independent of the cable (Fig. 20), the backstays will be straight and $f_1=0$. Consequently, all terms containing f_1, y_1, $n_1=\frac{f_1}{l_1}$, or $v=\frac{f_1}{f}$, will vanish in Eqs. (177) to (218), inclusive.

On account of the continuity of the trusses, however, each span will be affected by loads in the other spans.

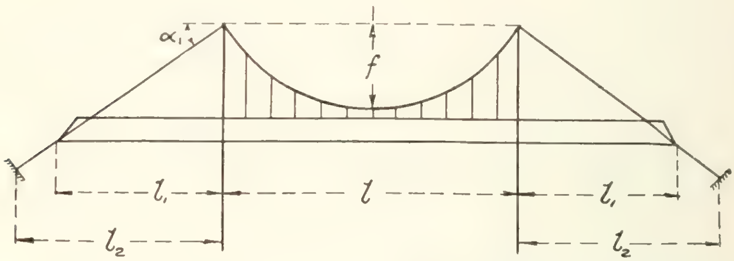


FIG. 20.—Continuous Stiffening Truss with Straight Backstays. (Type $0F$).

The denominator of the expression for H, Eq. (203), will become,

$$N = \tfrac{8}{5} - 2e + \frac{E}{E_c}\frac{3I}{Af^2}\frac{l'}{l}(1+8n^2) + \frac{6I}{A_1f^2}\frac{E}{E_c}\frac{l_2}{l}\cdot\sec^3\alpha_1, \qquad (219)^*$$

where e, the factor of continuity, now has the value,

$$e = \frac{2}{3+2ir}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (220)$$

Equation (183), for bending moments in the side spans, will become,

$$M = M_0 + \frac{x_1}{l_1}M_{1,2} + H\frac{x_1}{l_1}\cdot ef, \quad . \quad . \quad . \quad . \qquad (221)$$

---

* See Footnote to Eq. (127).

and Eq. (189), for shears in the side spans, will become,

$$V=V_0\pm\frac{M_{1,2}}{l_1}+H\cdot\frac{ef}{l_1}. \qquad (222)$$

For a concentration P_1 in either side span, Eq. (205) becomes,

$$H=-\frac{1}{2N\cdot n}\cdot ir^2e(k_1-k_1^3)\cdot P_1. \qquad (223)$$

For a uniform load covering both side spans, Eq. (207) becomes,

$$H=-\frac{ir^3e}{4N\cdot n}\cdot p_1l. \qquad (224)$$

For a uniform load in either side span, covering a length k_1l_1 from the free end, Eq. (209) becomes,

$$H=-\frac{ir^3e}{8Nn}(2-k_1^2)\cdot k_1^2\cdot p_1l. \qquad (225)$$

For a uniform load covering all three spans, Eq. (211), for the bending moments in the side spans, becomes

$$\text{Total } M=\tfrac{1}{2}px_1(l_1-x_1)-\frac{ex_1}{l_1}(\tfrac{1}{8}pl^2-Hf). \qquad (226)$$

Equation (216), for temperature moments in the side spans, becomes,

$$M_t=+H_t\cdot\frac{x_1\cdot ef}{l_1}, \qquad (227)$$

and Eq. (218), for the shears, becomes,

$$V_t=+H_t\cdot\frac{ef}{l_1}. \qquad (228)$$

Section VII.—Braced-chain Suspension Bridges

32. Three-hinged Type (3*B*).—The three-hinged type of braced-chain suspension bridge is statically determinate. The suspension system in the main span is simply an inverted three-hinged arch. The equilibrium polygon for any applied loading will always pass through the three hinges. The H-influence line for vertical loads reduces to a triangle whose altitude, if

the crown-hinge is at the middle of the span and if the corresponding sag is denoted by f, is,

$$H=\frac{l}{4f}. \qquad (229)$$

The determination of the stresses is made, either analytically or graphically, exactly as for a three-hinged arch.

Figure 21 shows the single-span type, in which the backstays are straight (Type $3BF$). If the lower chord is made to coincide with the equilibrium polygon for dead load or full live load, the stresses in the top chord and the web members will be zero for such loading conditions. These members will then be stressed only by partial or non-uniform loading. Under partial loading,

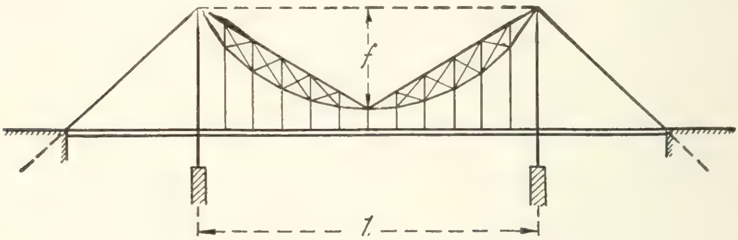


FIG. 21.—Three-hinged Braced Chain with Straight Backstays. (Type $3BF$).

the equilibrium polygon will be displaced from coincidence with the lower chord: where it passes between the two chords, both will be in tension; where it passes below the bottom chord, this member will be in tension and the top chord will be in compression. If the curve of the bottom chord is made such that the equilibrium polygon will fall near the center of the truss or between the two chords under all conditions of loading, the stresses in both chords will always be tension.

Figure 22 shows the three-hinged braced-chain type of suspension bridge provided with side spans (Type $3BS$). The stresses in the main span trusses are not affected by the presence of the side spans, and are found as outlined above. The stresses in the side spans are found as for simple truss spans of the same length, excepting that there must be added the stresses due to

the top chord acting as a backstay for the main span. This top chord receives its greatest compression when the span in question is fully loaded, and its greatest tension when the main span is fully loaded.

Temperature stresses and deflection stresses in three-hinged structures are generally neglected.

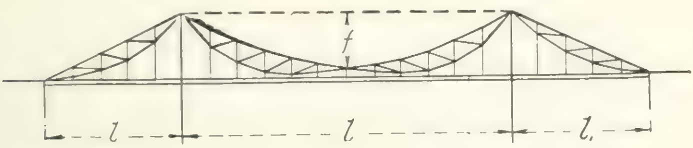


FIG. 22.—Three-hinged Braced Chain with Side Spans. (Type $3BS$).

33. Two-hinged Type ($2B$). This system (Fig. 23) is statically of single indetermination with reference to the external forces, so that the elastic deformations must be considered in determining the unknown reaction.

The structure is virtually a series of three inverted two-

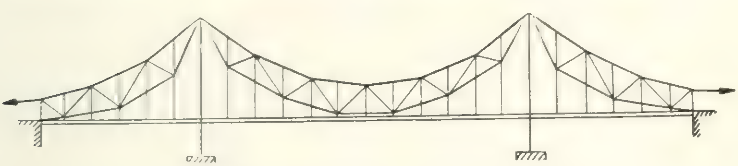


FIG. 23.—Two-hinged Braced Chain with Side Spans (Type $2BS$).

hinged arch trusses, having a common horizontal tension H resisted by the anchorage.

The value of H may be determined by the same method as was used for writing Eqs. (116) and (117). In this case, the general equation for H takes the form,

$$H=\frac{\Delta}{\delta}=-\frac{\sum\frac{Zul}{EA}}{\sum\frac{u^2l}{EA}}, \quad . \quad . \quad . \quad . \quad (230)$$

where Z denotes the stresses in the members for any external loading when $H=0$ (i.e., when the system is cut at the anchorages); u denotes the stresses produced under zero loading when $H=1$; l denotes the lengths of the respective members and A their cross-sections. The summations embrace all the members in the entire system between anchorages.

The stress in any member is given by adding to Z the stress produced by H, or,

$$S=Z+H\cdot u. \quad \ldots \ldots \quad (231)$$

For a rise in temperature, the elastic elongations $\frac{Zl}{EA}$ are replaced by thermal elongations ωtl, and Eq. (230) becomes,

$$H_t=\frac{\Delta}{\delta}=-\frac{\Sigma\omega tul}{\sum\frac{u^2l}{EA}}. \quad \ldots \ldots \quad (232)$$

For uniform temperature rise in all the members, Eq. (232) may be written,

$$H_t=-\frac{\omega tL}{\sum\frac{u^2l}{EA}}, \quad \ldots \ldots \quad (233)$$

where L is the total horizontal length between anchorages.

Equations (230) to (233) may also be used for the ordinary types of suspension bridge with straight stiffening truss (Types $2F$ and $2S$) if the summations are applied to the individual members of the stiffening truss and to the segments of the cable between hangers. (The hangers and towers may also be included.) This will give more accurate results than the ordinary method, as it takes into account the varying moments of inertia of the stiffening truss and any variations from parabolic form of cable.

A graphic method of determining H is to find the vertical deflections at all the panel points produced by a unit horizontal force ($H=1$) applied at the ends of the system. The resulting deflection curve will be the influence line for H. If the ordinates of this curve are divided by the constant δ (the horizontal displacement of the ends of the system produced by the same force

$H = 1$), they will give directly the values of H produced by a unit vertical load moving over the spans.

34. Hingeless Type ($0B$). This type of suspension bridge (Fig. 24) is threefold statically indeterminate, the redundant unknowns being the horizontal tension H and the moments at the towers. Instead, the stresses in any three members, such as the members at the tops of the towers and one at the center of the main span, may be chosen as redundants. Let the stresses in the three redundant members under any given loading be denoted by X_1, X_2, X_3. When these three members are cut, the structure is a simple three-hinged arch; in this condition, let Z denote the stresses produced by the external loads, and let u_1, u_2 and u_3 denote the stresses produced by applying internal

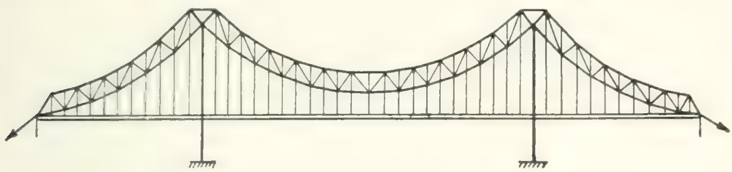


FIG. 24.—Hingeless Braced Chain Suspension Bridge. (Type $0B$.

forces $X_1 = 1$, $X_2 = 1$, and $X_3 = 1$. Then, when the three redundants are restored, the stress in any member will be,

$$S = Z + X_1 u_1 + X_2 u_2 + X_3 u_3. \qquad (234)$$

The restoration of the redundant members must satisfy the three conditions,

$$\sum \frac{S u_1 l}{EA} = 0, \quad \sum \frac{S u_2 l}{EA} = 0, \quad \text{and} \quad \sum \frac{S u_3 l}{EA} = 0, \qquad (235)$$

which, with the aid of Eq. (234), may be written:

$$\left.\begin{aligned}
\sum \frac{Z u_1 l}{EA} + X_1 \sum \frac{u_1^2 l}{EA} + X_2 \sum \frac{u_1 u_2 l}{EA} + X_3 \sum \frac{u_1 u_3 l}{EA} &= 0 \\
\sum \frac{Z u_2 l}{EA} + X_1 \sum \frac{u_1 u_2 l}{EA} + X_2 \sum \frac{u_2^2 l}{EA} + X_3 \sum \frac{u_2 u_3 l}{EA} &= 0 \\
\sum \frac{Z u_3 l}{EA} + X_1 \sum \frac{u_1 u_3 l}{EA} + X_2 \sum \frac{u_2 u_3 l}{EA} + X_3 \sum \frac{u_3^2 l}{EA} &= 0
\end{aligned}\right\} . \qquad (236)$$

The redundant members are to be included in these summations.

The solution of these three simultaneous equations will yield the three unknowns X_1, X_2 and X_3, and their substitution in Eq. (234) will give the stresses throughout the structure.

CHAPTER II

TYPES AND DETAILS OF CONSTRUCTION

1. Introduction.—The economic utilization of the materials of construction demands that, as far as possible, the predominating stresses in any structure should be those for which the material is best adapted. The superior economy of steel in tension and the uncertainties involved in the design of large-sized compression members point emphatically to the conclusion that the material of long-span bridges, for economic designs, must be found to the greatest possible extent in tensile stress. This requirement is best fulfilled by the suspension-bridge type.

The superior economy of the suspension type for long-span bridges is due fundamentally to the following causes:

1. The very direct stress-paths from the points of loading to the points of support.
2. The predominance of tensile stress.
3. The highly increased ultimate resistance of steel in the form of cable wire.

For heavy railway bridges, the suspension bridge will be more economical than any other type for spans exceeding about 1500 feet. As the live load becomes lighter in proportion to the dead load, the suspension bridge becomes increasingly economical in comparison with other types. For light highway structures, the suspension type can be used with economic justification for spans as low as 400 feet.

Besides the economic considerations, the suspension bridge has many other points of superiority. It is light, aesthetic, graceful; it provides a roadway at low elevation, and it has a low center of wind pressure; it dispenses with falsework, and is easily constructed, using materials that are easily transported;

there is no danger of failure during erection; and after completion, it is the safest structure known to bridge engineers.

The principal carrying member is the cable, and this has a vast reserve of strength. In other structures, the failure of a

FIG. 25.—Brooklyn Bridge.
East River, New York. Span 1595½ feet. Completed 1883.

single truss member will precipitate a collapse; in a suspension bridge, the rest of the structure will be unaffected. In the old Niagara Railway Suspension Bridge (built 1855), the chords of the stiffening truss were broken (due to overloading) and repaired repeatedly, without interrupting the railroad traffic.

STIFFENED SUSPENSION BRIDGES	Stiffening truss	Non-continuous	Center hinge (3-hinged)	Side spans free,	$3F$,	
				Side spans suspended,	$3S$,	Brooklyn Br., N. Y.
			No center hinge (2-hinged)	Side spans free,	$2F$,	Williamsburg Br., N. Y.
				Side spans suspended,	$2S$,	Manhattan Br., N. Y.
		Continuous	Center hinge (1-hinged)	Side spans free,	$1F$,	Ohio River Br., Cincinnati
				Side spans suspended,	$1S$,	
			No center hinge (0-hinged)	Side spans free,	$0F$,	Elizabeth Br., Budapest
				Side spans suspended,	$0S$,	Rondout Br., Kingston
	Braced-chain B	Three hinged $3B$	Parabolic lower chord $3BL$	Side spans free,	$3BLF$,	Point Br., Pittsburgh
				Side spans suspended,	$3BLS$,	
			Parabolic upper chord $3BU$	Side spans free,	$3BUF$,	Youngstown, Ohio
				Side spans suspended,	$3BUS$,	Grand Ave., St. Louis
			Parabolic center line $3BC$	Side spans free,	$3BCF$,	
				Side spans suspended,	$3BCS$,	2nd Quebec Design
		Two hinged $2B$	Parallel chords $2BP$	Side spans free,	$2BPF$,	
				Side spans suspended,	$2BPS$,	Seventh St., Pittsburgh
			Varying depth $2BV$	Side spans free,	$2BVF$,	
				Side spans suspended,	$2BVS$,	1st Quebec Design
			Horizontal lower chord $2BH$	Side spans free,	$2BHF$,	
				Side spans suspended,	$2BHS$,	
		Hingeless $0B$	Parallel chords $0BP$	Side spans free,	$0BPF$,	
				Side spans suspended,	$0BPS$,	Hudson River Br.

Special Symbols: E = Eyebar Chains. D = Diagonal Stays.

There are two main classes of suspension bridges: those with suspended stiffening truss (Figs. 25 to 36), and those with overhead braced-chain construction (Figs. 37 to 41). For purpose of reference, there is given here (page 71) a comprehensive system of classification of suspension bridges, with mnemonic type symbols and outstanding examples.

2. Various Arrangements of Suspension Spans.—The simplest form of suspension bridge is a single span (Type 2*F* or 3*F*)

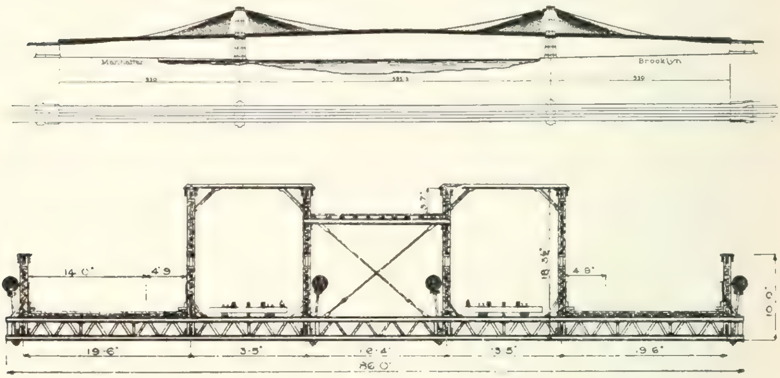


FIG. 26.—Brooklyn Bridge.
(Type 3*SD*).

Elevation, Plan, and Cross-section.

with the cable carried past the towers as diagonal backstays (Figs. 27, 29). If side spans are added (Fig. 28), they are independent of the cable and of the main span. The single-span suspension bridge may be built either with or without a stiffening truss (Fig. 27).

The next form is the bridge having three suspended spans (Types 0*S*, 1*S*, 2*S*, 3*S*). In this form, stiffening trusses (or girders) are indispensable. Only two towers are required, and each side span is about one-half the length of the main span (Figs. 10, 17, 25, 30, 33, 35).

If the main span is provided with a center hinge (in addition to end hinges), the three-span structure becomes statically determinate (Type 3*S*, Fig. 26). The side spans are suspended

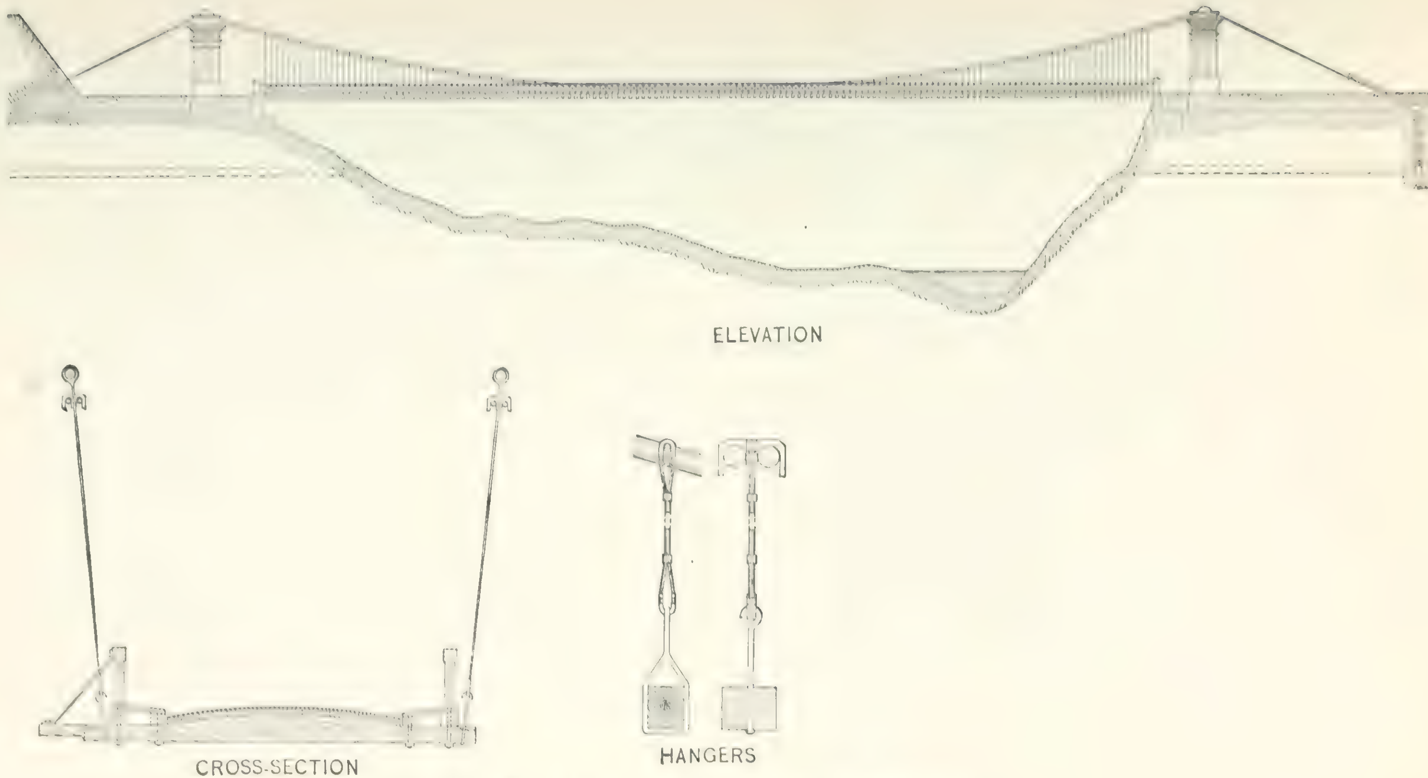


Fig. 27.—Freiburg Suspension Bridge (Switzerland).

Span 870 feet. Unstiffened. Built 1834. Two cables added 1881.

from the cable, but carry their loads as simple beams without affecting the stresses in the cable or in the main span; on the other hand, any load in the main span or tension in the cable will produce relieving stresses in the side spans.

Multiple-span suspension construction, with more than two towers, is not efficient; the great economy of the suspension type is lost. As the number of spans increases, the value of the cable tension H is proportionately reduced, and more of the load is thrown upon the stiffening trusses; the bending moments and the deflections are thus greatly increased. Examples of this type are the Lambeth Bridge, London, with three equal spans of 280 feet; and the Seventh St. Bridge, Pittsburgh, having two main spans of 330 feet and two side spans of 165 feet each.

Multiple-span suspension designs have been proposed with the intermediate piers serving as anchorages for adjoining spans. This has both economic and aesthetic disadvantages.

A suspension bridge of two spans with a single tower would not be economical. The tower would have to be twice the normal height to give the desired sag-ratio for the cables.

3. Wire Cables *vs.* Eyebar Chains.—One of the first questions to be decided in the design of a suspension bridge is the choice between a wire cable and a chain of eyebars (or flats) for the principal carrying member. The latter enables the bracing for the prevention of deformation under moving load to be incorporated in the suspension system; the other ordinarily requires a separate stiffening truss for the reduction of these deflections.

The earliest suspension bridges were built with chains. At first (1796) forged wrought-iron links were employed; then (1818) wrought-iron eyebars were introduced; and later (1828) open-hearth steel eyebars came into use. John A. Roebling established the use of wire cables (about 1845); and since his time, wire cables have been used in practically all suspension bridges. (Two notable exceptions are the Elizabeth Bridge at Budapest [Fig. 34] and the Rhine Bridge at Cologne [Fig. 17]).

In Lindenthal's first Quebec Design (Fig. 39), and in his 1894 design for the projected Hudson River Bridge, he pro-

posed building the braced chains of pin-connected wire links; such construction would have the advantages of accurate work and close inspection in the shop, rapid erection, and possibility of varying the cable-section as required. Thus far there has been no opportunity, however, of demonstrating the feasibility of combining the overhead bracing system with a cable or chain of wire.

The economic comparison of wire cable and eyebar construction rests on the following considerations: Steel wire with an elastic limit of 150,000 pounds per square inch costs but twice as much as nickel-steel eyebars with one-third the elastic limit. The eyebar heads and pins add about 20 per cent to the weight of the chain. The wire cable is self-supporting during erection and all the problems involved have been worked out and successfully demonstrated. The eyebars, on the other hand, unless expensive falsework is used, would require temporary supporting cables; and the manufacture and erection of eyebars of suitable size for very long spans present many unsolved difficulties.

The following points have

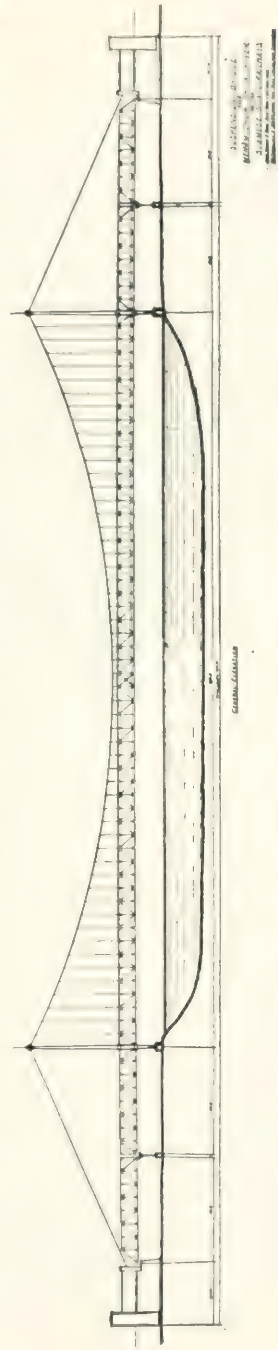


FIG. 28.—Suspension Bridge for the Siamese State Railways. (Type 2*FE*).
Span 900 feet. Eyebar Chains. Design by D. B. Steinman and F. H. Frankland, 1921.

been advanced in favor of chain construction: The section of an eyebar chain may be varied with the stress, whereas the entire wire cable must have the maximum section. The possibility of corrosion in wire cables (if not properly protected) and of unequal stressing of the wires (if not carefully strung) are further arguments for adopting eyebars. Finally, pin-connected eyebars permit speedier erection, especially in spans which would require large-sized cables.

Comparative designs have shown that, although the eyebar chain is 2 to $2\frac{1}{2}$ times as heavy as the wire cable, the difference in cost is generally very small. Where two designs are of equal cost, the heavier bridge is to be preferred as giving a more rigid structure. Greater weight, if it does not increase the cost, is an advantage in a bridge, as it serves to increase the rigidity and the life of the structure.

With present materials and prices, chain construction becomes more expensive than wire cables at about 1000-foot span. (See Fig. 28.) The development of high alloy steels at a sufficiently low unit price may, however, enable eyebar construction to displace wire cables even in the longest spans.

For the proposed Hudson River Bridge of 3240 feet span (Frontispiece and Fig. 41), it was found that the adoption of the overhead bracing system instead of a suspended stiffening truss yielded a saving which greatly outweighed the extra cost of employing eyebars instead of wire cables.

4. Methods of Vertical Stiffening.—On account of the deformations and undulations under moving load, unstiffened suspension bridges should not be used except for footbridges.

If no stiffening truss is provided, the distortions and oscillations of the cable may be limited by using a small sag-ratio; by making the floor deep and continuous; or by employing a latticed railing as a stiffening construction (Figs. 27, 33).

Another method of stiffening the suspension bridge is by the introduction of diagonal stays between the tower and the roadway (Fig. 26). These, however, have the disadvantage of making the stress-action uncertain, and of becoming either overstressed

or inoperative under changes of temperature; moreover, they introduce unbalanced stresses in the towers.

In recent French construction, diagonal stays are utilized, but the redundancy of members is more or less remedied by omitting the suspenders near the towers (Fig. 29). The indeterminateness is thus relieved, and the cable stress is reduced. This arrangement may be used to advantage in the reconstruction of weak suspension bridges.

A different method of vertical stiffening, known as the Ordish–Lefeuvre System, dispenses with cable and suspenders; it consists of diagonal stays running from the tops of the towers and meeting at a number of points along the span, so as to provide a triangular suspension for each point. These diagonal stays are

FIG. 29.—Suspension Bridge at Cannes-Ecluse.
(Type *2FD*).

Over the Yonne River (France). Span 760 feet. Built 1900. Wire Rope Cables. Diagonal Stays.

held straight by hangers from a light catenary cable overhead. This system was used for a bridge at Prague (1868) and for the Albert Bridge in London (1872). It proved to be uneconomical and unsatisfactory. A modified form, known as the Gisclard System, was devised for a bridge at Villefranche in 1908 and has since been copied for several other spans in France, despite its structural and aesthetic drawbacks.

Practically all modern suspension bridges are stiffened by means of a truss construction, either separate (Figs. 25–36) or incorporated in the cable system (Figs. 37–41). The different types of stiffening trusses and braced-chain designs will be discussed in separate sections.

5. Methods of Lateral Stiffening.—To give the structure lateral stiffness against wind forces, the most effective means is a

complete system of lateral bracing. If this bracing is in the plane of the top or bottom chords of the stiffening truss, these chords may act as members of the lateral systems (Figs. 30, 36); otherwise, separate wind-chords must be provided (Figs. 38, 39, 40).

The wind bracing just described is sometimes supplemented by land-ties or wind-anchors, i.e., ropes connecting points on the roadway to the piers (Fig. 38) or to points on shore. A horizontal suspension system may thus be formed (Fig. 38).

Another device for securing lateral stiffness is by building the cables and suspenders in inclined planes (Figs. 27, 30). This "cradling" of the cables, however, does not appreciably increase the lateral stability of the structure if there is but one cable on each side. If two or more cables of different inclinations are provided on each side (Figs. 26, 32), lateral stability is secured, but at the sacrifice of equal division of cable stresses.

Cradled cables, even if they do not prevent lateral deflection, will help to bring the resulting oscillations more quickly to rest—an important desideratum in long spans.

6. Comparison of Different Types of Stiffening Truss.—As a result of a comparative estimate of different types of stiffened suspension bridge, the following relative weights of cable and truss (in main span) were obtained.

Type	Relative Weight of Cable	Relative Weight of Truss	Relative Combined Weight
$0S$	103	103	103
$1S$	111	107	109
$2S$	100	100	100
$2F$	101	89	95
$3F$	102	82	92

The hingeless type ($0S$) (Fig. 17), gives the most rigid structure, as a result of the continuity of the stiffening truss over the towers. The deflections will be about $\frac{1}{3}$ less than those of a two-hinged stiffening truss of the same dimensions. This

greater rigidity is secured at an expense of only 3 per cent increase in the cost of the structure.

The one-hinged type (*1S*) is the least desirable construction. It has the highest cable stresses and chord stresses of any type of stiffening truss. It will cost more than any other type; and the large variation in chord stresses, the abrupt reversals of shear, and the lack of rigidity are serious disadvantages.

The two-hinged types (*2S* and *2F*) are widely used and are probably the most efficient types, all things considered. They are more economical than the continuous types, and are simpler to figure. They are far more rigid than type *3F*. The hinges are located in the towers, where they are least objectionable.

Comparing types *2S* and *2F*, we find that leaving the side spans free (straight backstays, Type *2F*) (Fig. 31) reduces the bending moments in the main span. The main-span truss weight is thus reduced by about 11 per cent, without sensibly affecting the cable weight. For lightness of truss, type *2F* is exceeded only by the three-hinged suspension bridge. Type *2F* is also more rigid than type *2S*.

Suspending the side spans (Type *2S*) (Figs. 30, 35) makes the cable more flexible, thus throwing more load on the stiffening truss. As a result, about 11 per cent is added to the weight of the truss in the main span, and the cable stress is slightly relieved. The increase in cost of the main span is generally more than offset, however, by the saving in the side spans as a result of their suspension. Without any addition to its weight, the cable relieves the side spans of their full dead load and nearly all of their live load. Type *2S* will consequently be more economical than *2F* or *3F* unless the conditions at the site are favorable to cheap, independent approach spans (Fig. 31). Another advantage of suspended side spans is the dispensing with falsework for their erection (Fig. 50).

The three-hinged type (*3F*) is determinate for calculations. The addition of the center hinge slightly increases the cable stress, but effects a small reduction in weight of stiffening truss. This type is little used on account of its lack of rigidity and other disadvantages arising from the hinge at mid-span. Inter-

mediate hinges are troublesome and expensive details, particularly in long spans; besides augmenting the deflections, they cause sudden reversals of shear under moving load, and constitute a point of weakness and wear in the structure. There is also a large waste of material in the minimum chord sections near the hinges which, in many cases, will offset the theoretical reduction in the weight of the stiffening truss. Furthermore, a center hinge conduces to a serious distortion of the cable from the ideal parabolic form, with a resulting overloading of some of the hangers. In the case of the Brooklyn Bridge (Fig. 25), the center hinge or slip joint has caused excessive bending stresses in the cable at that point, and the breaking of the adjacent suspenders; 120 suspenders near the hinge had to be replaced by larger ropes.

7. Types of Braced-Chain Bridges. A stiffening construction incorporated in the suspension system may be used instead of the straight stiffening truss at roadway level. The former construction, as a rule, involves the use of eyebar chains instead of wire cables (Figs. 37, 38, 40, 41).

A braced-chain suspension bridge is virtually an inverted arch in which the ends are capable of restricted horizontal movements. The stresses are the same as those in an arch, but with opposite signs; the principal stress is tension, instead of compression.

Braced-chain bridges may be classified as to the number of hinges ($0B$, Fig. 41; $2B$, Fig. 39; $3B$, Fig. 38); or as to outline of the suspension system (Parabolic Top Chord, Figs. 37, 39; Parabolic Bottom Chord, Fig. 38; Parabolic Center Line, Fig. 40; Parallel Chords, Fig. 41).

If the suspension system has a parabolic top chord and a straight bottom chord (Type $2BH$; Type $3BUH$, Fig. 37) it corresponds to a spandrel braced arch. The Lambeth Bridge, London, is an example. The top chord, like a cable, carries the entire dead load. If the live load is not too great in proportion, the top chord will never have its tensile stresses reversed; it may then be built as a flexible cable (Lambeth Bridge) or chain (Frankfort Bridge, Fig. 37). The bottom chord members suffer reversals of stress, hence they must be built as compression

members. For erection, the diagonals should be omitted until all the dead load and one-half the live load are on the structure at mean temperature; this procedure will minimize the extreme stresses in bottom chord and web members. The advantage of making the bottom chord straight is to save hangers and extra wind chords.

To avoid having very long diagonals near the ends of the span, the bottom chord may be bent up toward the towers (Type $2BV$, Fig. 39). This construction has the advantage of maximum truss depth near the quarter-points where the bending moments are also a maximum. The main part of the lower chord remains at the roadway level, thereby saving hangers and extra wind chords over that length.

If the bottom chord is made parabolic, it becomes the principal carrying member. This outline is best adapted for three-hinged systems (Type $3BL$). A notable example is the Point Bridge at Pittsburgh (Fig. 38). In this structure, the top chord consists of two straight segments, intersecting the bottom chord at ends and center. Since the bottom chord is the equilibrium curve for dead load, there are no dead-load stresses in the top chords or in the web members. The top chords must be made stiff members, as they are subject to reversals of stress. This form of suspension bridge (Type $3BL$) is statically determinate and easily figured. It avoids the use of long diagonals required in the spandrel braced types ($2BV$, Fig. 39; $2BH$; $3BUH$, Fig. 37), but it requires the addition of longitudinal and lateral stiffening in the roadway.

Instead of being straight lines (Fig. 38), the two top-chord segments may be curved. In a system proposed by Eads, they are made convex upward.

To avoid reversals of stress in the chord members, a form known as the Fidler Truss may be used. In this form (Type $3BC$), both chords are concave upward; and the line midway between top and bottom chords is made parabolic, so that the two chords will have equal tensions under dead load and uniform live load. An example of this form is Lindenthal's Second Quebec Design (Fig. 40). The outlines of the chords are obtained

by superimposing the two equilibrium curves for total dead load plus live load covering each half of the span in turn.

For two-hinged systems (Type $2BF$) a crescent-shaped truss may be used. The top and bottom chains meet at common supports on the towers, where they are connected to single backstays. There are no examples of this type.

If the top and bottom chains are kept parallel, we have either Type $2BP$ or Type $0BP$ (Fig. 41), according as the truss bracing is interrupted or continuous at the tower. Both of these types are indeterminate, and may involve some uncertainty of stress distribution. Unless the tower and anchorage details are properly worked out, there is danger of one of the parallel chains becoming overstressed or inoperative. Examples of these types are Lindenthal's Seventh St. Bridge at Pittsburgh (Type $2BP$) and his Hudson River Bridge design (Type $0BP$, Fig. 41).

An important advantage of the braced-chain system of construction over the straight stiffening truss is the greater flexibility of outline, with the possibility of varying the truss depths for maximum efficiency. By having the greatest depth of the bracing at the quarter points of the span, where the maximum moments occur, the stiffness of the bridge with a given expenditure of material is greatly increased; and by using a shallow depth along the middle third of the span, the temperature stresses are reduced.

The braced-chain construction (Types $2BV$, $2BH$, or $3BU$) saves one chord of the truss, as the cable itself forms the upper chord.

Advantages of the suspended stiffening truss (Figs. 25–36) are more graceful appearance, dispensing with extra wind chords, lower elevation of surfaces exposed to wind, less live-load effect on hangers and cables, simpler connections, easier and safer erection.

In addition, the braced-chain and suspended-truss types carry with them the respective advantages of eyebars and wire cables, unless the practically untried combination of overhead bracing with wire cables is adopted.

8. Economic Proportions for Suspension Bridges.—The minimum ratio of side spans to main span is about $\frac{1}{4}$ for straight

backstays, and about $\frac{1}{2}$ for suspended side spans. Shorter ratios tend to make the stresses or sections in the backstays greater than in the main cable. The length of side span is also controlled by existing shore conditions, such as relative elevations and suitable anchorage sites.

The economic ratio of sag to span of the cable between towers is about $\frac{1}{9}$ if the backstays are straight and about $\frac{1}{8}$ if the side spans are suspended. (See the author's book "Suspension Bridges and Cantilevers.") For light highway and foot-bridges, the sag-ratio may be made as low as $\frac{1}{10}$ to $\frac{1}{12}$.

For adequate lateral stiffness the width, center to center, of outer stiffening trusses should not be less than about $\frac{1}{24}$th of the span.

The economic depth of stiffening truss is about $\frac{1}{40}$th of the span (Fig. 31); although a shallower depth (Fig. 35), desirable for aesthetic reasons, will not materially augment the cost. For a railroad bridge, the truss depth (at the quarter points) should not be less than about $\frac{1}{45}$th of the span, or the deflection gradients will exceed 1 per cent. (See "Suspension Bridges and Cantilevers.") For highway bridges, the depth may be made as low as $\frac{1}{50}$th to $\frac{1}{60}$th of the span.

The economic span-limit for suspension bridges is about 3200 feet (Fig. 41). For greater span-lengths, the necessary outlay would not be warranted by traffic returns; but there are other returns, such as civic development and increase in realty values, to justify longer spans. Spans up to 5000 feet may be regarded as feasible.

9. Arrangements of Cross-sections.—The unit of suspension bridge design is the vertical suspension system, consisting of a cable (or group of cables) and the corresponding suspenders in a vertical (or slightly inclined) plane.

Since the suspension systems are above the roadway, their number is limited; they seldom exceed two (Figs. 27, 30, 32, 37–41). In wide bridges having a number of roadways, four suspension systems may be provided (Figs. 26, 36).

The main carrying element in each suspension system may be a single cable (Figs. 26, 36), two cables side by side (Figs. 27, 32),

two cables superimposed, or a group of cables or wire ropes (Figs. 27, 29, 30); or it may consist of a single chain of bars (Figs. 28, 37), two chains simply superimposed (Figs. 33, 34, 39), or two chains connected by web members to make a vertical stiffening system (Figs. 38, 40, 41).

There is generally one stiffening truss for each suspension system, and in the same plane; hence, there are ordinarily two (Figs. 30, 32, 37–41), and at most four stiffening trusses (Fig. 36). An exception is the Brooklyn Bridge (Fig. 26), having six stiffening trusses for four cables; this, however, has proved to be an unsatisfactory and inefficient arrangement.

Between the stiffening trusses are the roadways, generally on a single deck (Figs. 27, 30, 33, 34, 37–40). Sometimes two decks are provided, in order to provide the required number of traffic-ways (Figs. 26, 32, 36, 41). Where two decks are used, the railways are best placed below and the vehicular roadways above (Figs. 15, 41). In the Williamsburg Bridge (Fig. 32), a transverse truss is employed to carry the inside floorbeam reactions to the two outside suspension systems.

The floorbeams either terminate at their connections to the outer suspension systems (Figs. 26, 27, 30, 37–40); or they extend beyond as cantilever brackets to carry outside sidewalks or roadways (Figs. 32, 36, 41). The latter arrangement saves floorbeam weight; reduces width of towers, piers and anchorages; and helps in the separation of different traffic-ways. In very long spans, the first arrangement (with all roadways inside) may be necessary in order to maintain the requisite width between trusses for lateral stiffness.

Where there are four suspension systems, the floorbeams may be made continuous for greater stiffness (Fig. 36); or they may be provided with hinges to eliminate the indeterminateness.

10. Materials used in Suspension Bridges.—The stiffening trusses are generally built of structural steel, but nickel steel or other alloy steels may be used; and for minor structures, timber trusses have been employed.

The cables are generally made of galvanized steel wires having an ultimate strength of 200,000 to 230,000 pounds per

square inch, and an elastic limit as high as 150,000 pounds per square inch.

The suspenders are generally galvanized steel ropes. These are manufactured in diameters ranging from $1\frac{1}{4}$ to $2\frac{3}{4}$ inches, and have a tested ultimate strength given by $80{,}000 \times (\text{diameter})^2$.

In smaller bridges (Figs. 29, 30), the cables may be made of these galvanized steel ropes instead of parallel wires.

The towers are generally built of structural steel (Figs. 30, 31, 35, 37, 38, 41); although stone (Figs. 25, 27, 29, 33), concrete, and timber have been used.

Cast steel is used for all castings, such as saddles (Figs. 32, 33), cable bands (Figs. 32, 36), strand shoes (Figs. 32, 36), anchorage knuckles (Figs. 32, 33), and anchor shoes (Figs. 33, 37, 38). Suspender sockets (Figs. 32, 36) are made by drop-forging and machining.

If chains are adopted instead of wire cables, alloy steels may be advantageously employed. In a competition for a suspension bridge at Worms, the Krupp firm guaranteed nickel steel eyebars with an ultimate strength of 100,000 to 120,000, an elastic limit of 70,000 pounds per square inch, and an elongation of 15 per cent. For Lindenthal's Manhattan Bridge design (1902), nickel steel eyebars with an ultimate strength of 100,000 and with 20 per cent elongation were to be used. The chains for the Elizabeth bridge at Budapest (Fig. 34) were made of open-hearth steel with 70,000 to 80,000 ultimate strength and with 20 per cent elongation.

Under average conditions, the substitution of nickel steel affords a saving of 10 to 15 per cent in the cost of a chain or a stiffening truss.

11. Wire Ropes.—Galvanized steel ropes used for suspenders and for small bridge cables (Figs. 29, 30), are manufactured in diameters ranging from $1\frac{1}{4}$ to $2\frac{3}{4}$ inches. Each rope consists of 7 strands, each strand containing 7, 19, 37 or 61 wires; the wires are twisted into strands, in the opposite direction to the twist of the strands into rope, the angle of twist being about 18°. The weight of the rope in pounds per lineal foot is $1.68 \times (\text{diameter})^2$.

The strength of a twisted wire rope is less than the aggregate

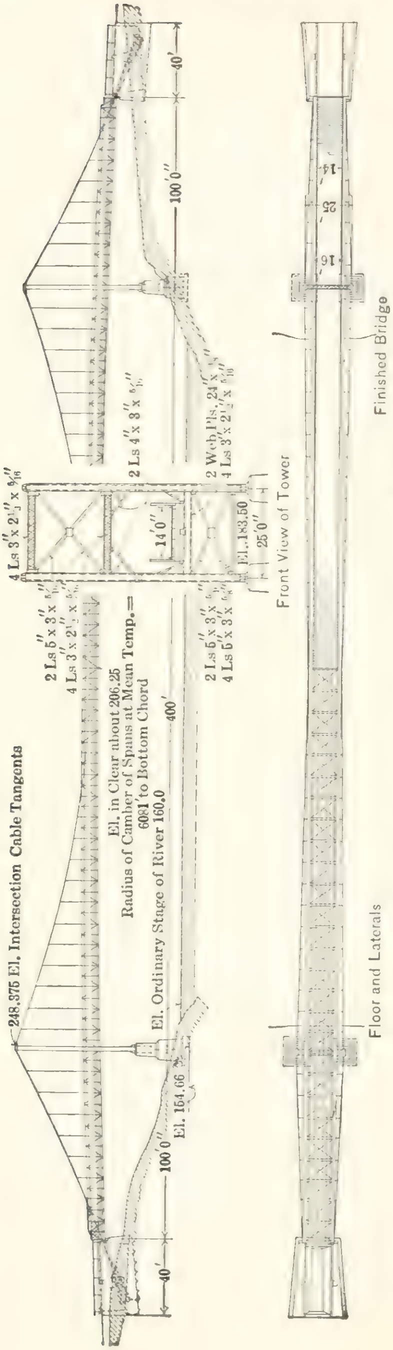


FIG. 30.—Suspension Bridge at Massena, N. Y. (Type 28).
Span 400 feet. Wire rope cables. Built by H. D. Robinson, 1912.

strength of the individual wires. The spiral wires are stressed about 4 or 5 per cent higher than the mean stress per square inch in the rope, and the center wire is stressed 15 per cent higher than the spiral wires. The tested ultimate strength of galvanized steel suspension bridge rope is given by 80,000 $\times$(diameter)^2.

When twisted wire ropes are used for cables, care must be observed, when applying the fundamental design formulas, to allow for the reduced elastic coefficient (E) of this material; it is only about $\frac{2}{3}$ of the value of E for structural steel.

The coefficient of elasticity (E) of a single rope strand with an angle of twist of 18° is 85 per cent of E for parallel wires, or about 24,000,000. The coefficient of elasticity (E) of a twisted wire rope composed of 7 or

more strands is 85 per cent of E for a single strand, or about 20,000,000.

Twisted wire ropes have a large initial stretch under load, on account of the spiral lay of the wires and strands. Consequently, at small loads, tests show a high rate of stretch yielding a modulus of elasticity (E) as low as 10,000,000. After the initial stretch has been taken up (at a unit stress of about 20,000 pounds per square inch), the rate of elongation is considerably reduced, yielding a value of 20,000,000 for the true elastic coefficient (E). The lower values of E (10,000,000 to 15,000,000) are to be used in estimating the dead-load elongation of the cable (if composed of wire ropes), and the higher value (20,000,000) should be used in figuring live-load and temperature stresses.

On account of the high and variable elongations, including the influence of time, suspenders and cables made of wire ropes should preferably be provided with screw and nut adjustments to regulate their lengths to the assumed deflections and elevations.

Cables may be built either of twisted wire ropes or of parallel wires. For long or heavy spans, parallel wire construction is best adapted; for light bridges, the use of twisted wire ropes may be more convenient and expeditious.

In cables formed of twisted wire ropes, the individual ropes are limited to 250 to 300 wires each, so as to avoid excessive stiffness and difficulty of handling; consequently, large cable sections require several such ropes.

A multi-strand cable may be formed of twisted strands surrounding a straight central strand; or of parallel strands united at intervals by clamps. Twisted strands ensure a more even division of load, except that the central strand carries a little more than its share; but the resulting cable suffers greater elongation under load. Moreover, since a twisted-strand cable must be erected as a unit, it is limited in weight and section. Equal stressing of parallel strands is dependent upon the efficiency of the clamps or bands in gripping them. An advantage of the parallel construction with bolted clamps is the ease of correcting overstress in individual strands and of replacing damaged strands. Clamping systems have been designed for

large groups of parallel ropes, to ensure unit stress action and to facilitate renewal of individual ropes; at the same time preserving ample spacing between the ropes to permit inspection and protection against rusting.

FIG. 31.—Williamsburg Bridge.
East River, New York. Span 1600 feet. Completed 1903.

A twisted wire cable of patent locked wire has been developed. In it the spiral wires have trapezoidal and Z-sections, locking together so as to leave practically no voids. The advantages

are compactness, smooth outer surface, firm gripping of the individual wires, and sealing against the entrance of moisture. Cables of this construction, ready to erect, have been made in single strands up to 800 tons tensile strength, and in seven strands up to 1500 tons.

The application of twisted ropes on a large scale involves problems requiring further study; whereas parallel wire construction has had ample satisfactory demonstration in the largest existing suspension bridges.

12. Parallel Wire Cables.—Parallel wire cables have the advantages of maximum compactness, maximum uniformity of stress in all the wires, and the easiest and safest connection of the cable to the anchorage. Twisted wire ropes are used for shorter spans, up to 600 or 700 feet, to save time in erection. Parallel wires are applicable to spans of any length, and will cost somewhat less than twisted ropes of the same strength; they will not stretch as much as twisted ropes, and will therefore keep more of the load off the stiffening truss. The only disadvantage of parallel wire cables is that they consume several weeks or months in erection.

A common size of wire for cables is No. 6 (Roebling gauge) which is 0.192 inch diameter and weighs 0.0973 pound per foot before galvanizing; after galvanizing, the diameter is about 0.195 inch, and the weight is practically $\frac{1}{10}$ pound per foot. The breaking strength of this wire at 220,000 pounds per square inch is 6400 pounds; the elastic limit at 150,000 pounds per square inch is 4350 pounds; the working stress at 75,000 pounds per square inch is 2180 pounds per single wire. Other common sizes of wire for cables are No. 7 (0.177 inch diameter) and No. 8 (0.162 inch diameter), recommended for shorter spans.

About 250 to 350 of these wires are treated as a single strand during erection. The cable consists of 7, 19, 37 or 61 of these strands. At the anchorages, the strands are looped around grooved shoes (Fig. 36) which are pin-connected to the anchorage eyebars (Fig. 32). For the rest of their length, the strands are compacted and bound to form a cylindrical cable of parallel wires.

For security against corrosion, the wire should be galvanized.

The only drawback is a reduction of about 7 per cent in the strength of the wire per square inch of final gross section (4 per cent actual reduction in the strength of the wire, and 3 per cent increase in gross section.)

The splicing of individual wires was formerly effected by wrapping the overlapping ends with fine wire. A more efficient splice (giving 95 per cent efficiency) is made by mitering the ends, threading them and connecting with small sleeve-nuts (Figs. 32, 36). Both methods have the disadvantage of disturbing the uniformity of the cable section. To reduce the number of such splices, the lengths of the individual wires as manufactured have been increased to 3300 feet. In some French bridges, the ends of the wires, after beveling, were joined by soldering; but the heat reduces the strength of the wire at the splice.

Besides using galvanized wires, additional protection is secured by providing a tight and continuous wire wrapping around the cable. Soft, annealed, galvanized wire of No. 8 or No. 9 Roebling gauge is commonly used. The function of this wrapping is to exclude moisture, to protect the outer wires, and to hold the entire mass of wires so tightly as to prevent chafing and ensure united stress action.

No record can be found of any rusting of wire cables employing either or both of the above described methods of protection.

13. Cradling of the Cables.—In the majority of suspension bridges, the main span cables do not hang vertically but in planes inclined toward one another, the inclination ranging from 1 : 20 to as much as 1 : 6. The stiffening trusses, however, are kept vertical. Even in designs with overhead bracing, the suspension systems have been cradled with inclinations ranging from 1 : 20 to 1 : 16.

Cradling is employed principally because it is supposed to augment the lateral stiffness of the structure; however, the advantage in this respect over vertical cables is but slight. With an inclination of 1 : 10, the increased resistance to lateral displacement is only 1 per cent. Moreover, with cradled cables any lateral displacement is accompanied by a tilting of the suspended structure, resulting in secondary stresses which are difficult to evaluate.

The following table gives data on the wire cables of the East River suspension bridges:

	Brooklyn (Fig. 25)	Williamsburg (Fig. 31)	Manhattan (Fig. 35)
Date	1876–1883	1898–1903	1903–1909
Main span	1595.5 ft.	1600 ft.	1470 ft.
Cable-sag	128 ft.	177 ft.	160 ft.
Total load, p. l. f.	35,500 lb.	75,000 lb.	104,000 lb.
Number of cables	4	4	4
Strands per cable	19	37	37
Wires per strand	278	208	256
Wire diameter	.165 in.	.192 in.	.195 in.
Total cross-section	533 sq. in.	888 sq. in.	1092 sq. in.
Cable diameter	$15\frac{3}{4}$ in.	$18\frac{5}{8}$ in.	$21\frac{1}{4}$ in.
Size of wrapping wire	.135 in.	....	.148 in.
Max. stress in cables	47,500 lb./sq. in.	50,300 lb./sq. in.	73,000 lb./sq. in.
Ult. strength of cables	160,000 lb./sq. in.	200,000 lb./sq. in.	210,000 lb./sq. in.

Resistance to lateral displacement is more significantly improved in proportion as the cable-sag is reduced and as the weight of the suspended structure is increased.

If cradling is adopted, the cables should not be wrapped until they are pulled into the final inclined position; otherwise serious local stresses will be produced in the cable wires near the saddles.

If the cables are cradled, the saddle reactions on the towers will be correspondingly inclined unless the backstay cables are made divergent; the latter arrangement (used in the Williamsburg Bridge, Fig. 32) increases the necessary width of the anchorage.

Cradling will be effective in producing lateral stiffness if two cables of different inclination are provided on each side (Figs. 26 and 32). This arrangement has the disadvantage of throwing unequal load on the cables when the wind acts on the structure; load is added to the cables inclined in the direction of the wind, and those inclined in the opposite direction are relieved of load.

14. Anchoring of the Cables.—Parallel wire cables are anchored by making the end of each strand in the form of a sling. With the wrapping omitted at the end of the cable, the free wires loop around a half-round, flanged casting called a strand shoe

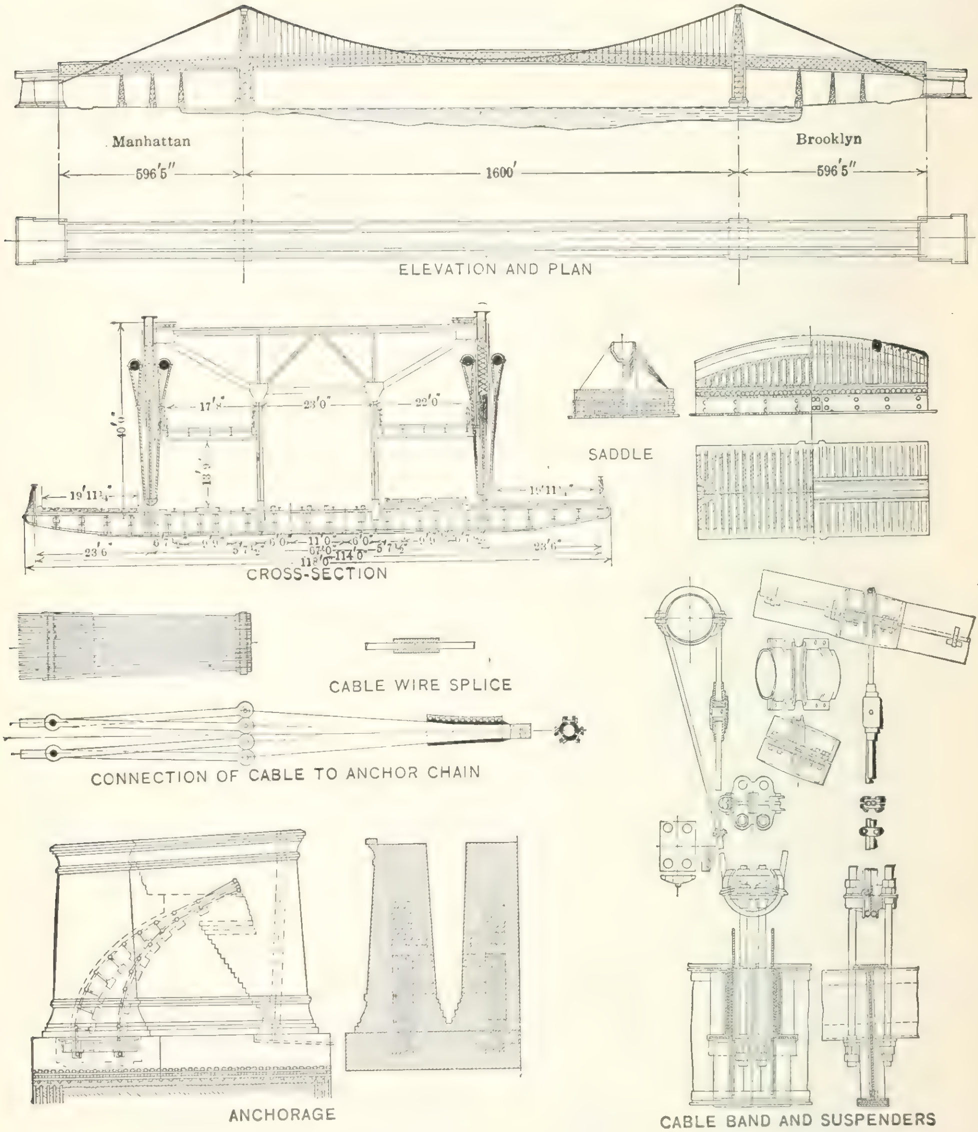


FIG. 32.—Williamsburg Bridge.
(Type 2*F*).

(Fig. 30), and then pass back into the strand. Large cables are divided for this purpose into 7, 19 or 37 strands; and such cables accordingly have 7, 19, or 37 strand shoes at each end. Steel pins pass through these shoes for connection to the anchorage eyebars (Fig. 32).

The strand shoes are grouped into a number of horizontal rows (generally 2 or 4), and the anchor chain divides into an equal number of branches to effect the connection (Fig. 32).

About 10 feet forward of the shoe, the two halves of a strand are combined into one; and all strands, before leaving the masonry, are squeezed into a round cable.

The shoes have slotted pin-holes which are provided with shim-blocks (Fig. 36) to permit regulation of the individual strands before combining into a cable.

Bending the wires around the shoe produces bending stresses exceeding the elastic limit; but the resulting stretching of the outer fibers redistributes the stress over the cross-section of the wire; with properly ductile steel wire, the strength at the loop is not materially impaired.

If the wire is very hard, or if the cable consists of wire ropes, a larger radius of curvature must be provided or some other form of connection must be used.

For wire ropes a larger shoe is used, with the end of the rope fastening into a socket after bending around the shoe.

The sling construction is avoided by setting the ends of the strands or ropes directly into steel sockets. After inserting the rope into the expanding bore of the socket, the wires are pried apart and spread with a point tool, and the intervening space filled with fusible metal (preferably molten zinc). Such socket connections are now made to develop the full strength of the rope. The sockets may be designed to bear directly against the under side of the anchor girder; or they may be threaded to receive the end of a rod which serves as a continuation of the strand.

15. Construction of Chains.—Chains may be constructed of horizontal flats piled together and spliced at intervals by means of friction clamps with bolted flanges. Suspenders are bolted

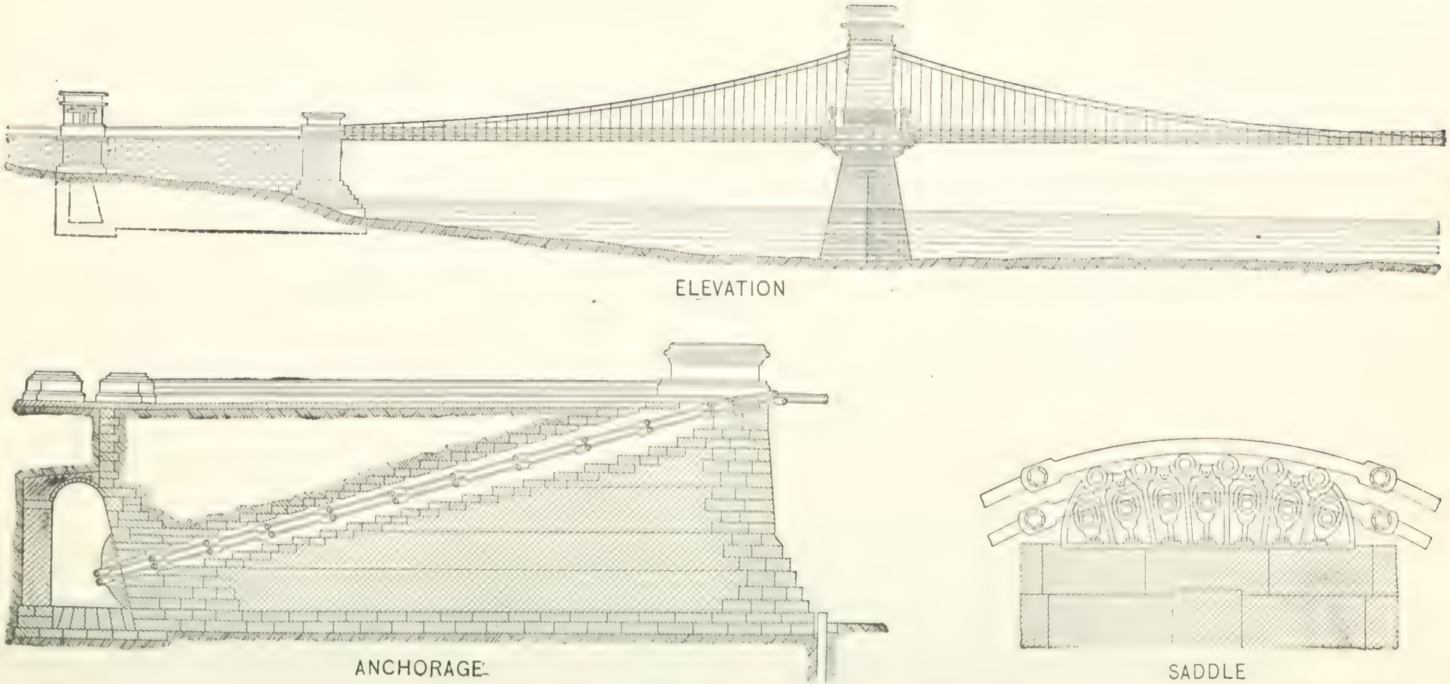


FIG. 33.—Clark's Bridge over the Danube at Budapest.
Span 663 feet. Eyebar Chains. Unstiffened. Completed 1845.

to these clamps. This laminated construction is subject to high secondary stresses from bending.

Chains may be constructed of closed links overlapping around connecting pins to which the hangers are attached (Fig. 39).

Chains may consist of eyebars or flats bored at their ends to receive pins (Figs. 33, 34, 38, 40). Generally, single-pin connections are used, and the number of bars alternates in successive panels. Otherwise, short two-pin connecting bars may be used, permitting the number of eyebars to be the same in adjacent panels.

In American practice (Figs. 38, 40), forged eyebars are used. In European practice (Figs. 17, 33, 34), the eyebars are made by welding or riveting, or by cutting from wide flats. The last is an extravagant procedure.

Where flats are used, the reduction of section by the pin-holes may be largely made up by riveting pin-plates at the ends of the bars.

Chains composed of vertical flats riveted together have been proposed, but the secondary stresses from bending would be very high.

For long spans, the chains would have large cross-sections, requiring pins of excessive length.

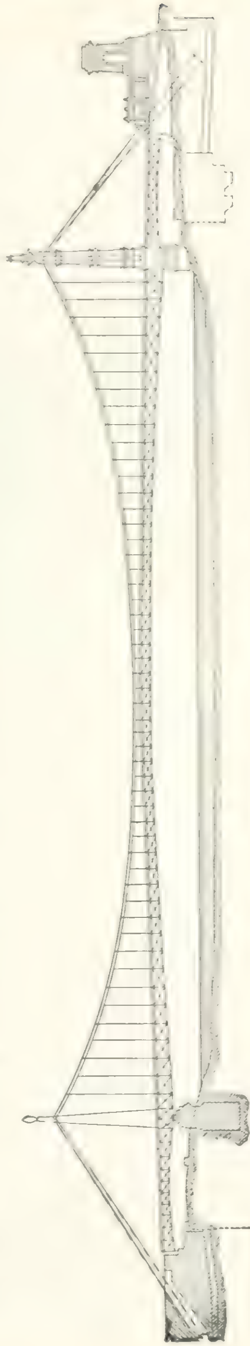


FIG. 34.—Elizabeth Bridge over the Danube at Budapest. (Type 0/FE).
Span 951 feet. Eyebar Chains. Completed 1903.

This is circumvented by using two chains, either side by side or superimposed; in the latter case, if the panels are not too short, successive hangers are connected alternately to the upper and lower chains (Figs. 33, 34).

A disadvantage of chain construction is unequal division of stress in the individual bars between two pins. This may be caused by inaccuracies in length, differences in temperature, variations in elastic modulus, bending of the pins, and eccentric suspender loading. The unequal stressing of the eyebars is frequently apparent on superficial examination or upon comparing the ringing pitch under hammer blows. Actual measurements (by comparing deflections under lateral test loads) have revealed varying stresses in a single group of eyebars ranging from 40 to 200 per cent of the mean stress.

16. Suspender Connections.—Cable Bands and Sockets.— The attachment of the suspenders to the cable is generally made by means of cast steel collars called cable bands (Figs. 26, 32, 36). The cable band may be an open ring with flanged ends to receive a clamping and connecting bolt (Fig. 26). More generally it is made in two halves with flanges (Figs. 32, 36). The band must grip securely to prevent slipping. The inside of the band should be left rough to minimize the tendency to slip on the cable; and space should be left between the flanges for taking up any looseness of grip, when necessary. A cam-clamping device has been proposed for automatically increasing the grip as load is applied through the suspender.

If the hangers are of rigid section, they are bolted to vertical flanges cast integral with the cable band for this purpose.

If rope suspenders are used, the cable band is cast with a groove or saddle to receive the rope which passes over it (Figs. 32, 36). On account of the varying slope of the cable, the grooves in the cable bands are at varying angles, requiring a number of different patterns. To avoid this, the bearing flange of the grooves may be made curved in elevation.

If the cable is used as a chord of an overhead bracing system, the rigid web members connect to the cable bands; and the latter must be made long enough, with ample clamping bolts, to

FIG. 35.—The Manhattan Bridge over the East River, New York.
Span 1470 feet. Completed, 1909.

develop the friction requisite to take up the chord increment of the web stresses. A tight layer of wire wrapping against the ends of the cable bands will add to their security against slipping.

The frictional grip (in pounds) attainable in a cable band, with maximum permissible stress in the bolts, is 70 to $100nd^2$, where n is the number and d is the diameter (in inches) of the clamping bolts. By this relation may be determined the number of bolts required to resist a given component parallel to the cable.

If the cable consists of a cluster of wire ropes, soft metal fillers should be inserted within the band to improve the grip and to exclude moisture.

The cable band should be designed so as to prevent the admission of moisture to the cable. The flanges should be designed for excluding rain, and the joints all around should be securely calked. The band should preferably be undercut at both ends for the insertion of the first few turns of the wire wrapping (Fig. 36).

The free ends of the suspender ropes are secured in sockets made of high-grade steel drop-forgings (Figs. 32, 36). The end of the rope is inserted into the expanding shell of the socket, the ends of the wires are spread apart and the interstices are filled with molten metal (preferably zinc) which will not shrink appreciably on cooling. This fastening of the end of the rope is found, by test, to be unaffected by the ultimate loads causing failure of the rope.

Closed sockets terminate in a closed loop with which other links can be engaged. Open sockets terminate in two parallel eye-ends to receive a bolt or pin for connection to other structural parts. Threaded sockets (Figs. 32, 36) are cylindrical and are threaded on the outside to receive adjusting and holding nuts; these sockets may be passed through truss chords or girder flanges, with the nuts bearing up against the lower cover plates of these members (Fig. 36).

Sockets are furnished by the wire rope manufacturers, either loose or fastened to the ropes.

17. Suspension of the Roadway.—The suspenders may consist of wire ropes (Figs. 26–28, 30–32, 36); or of rods, bars or

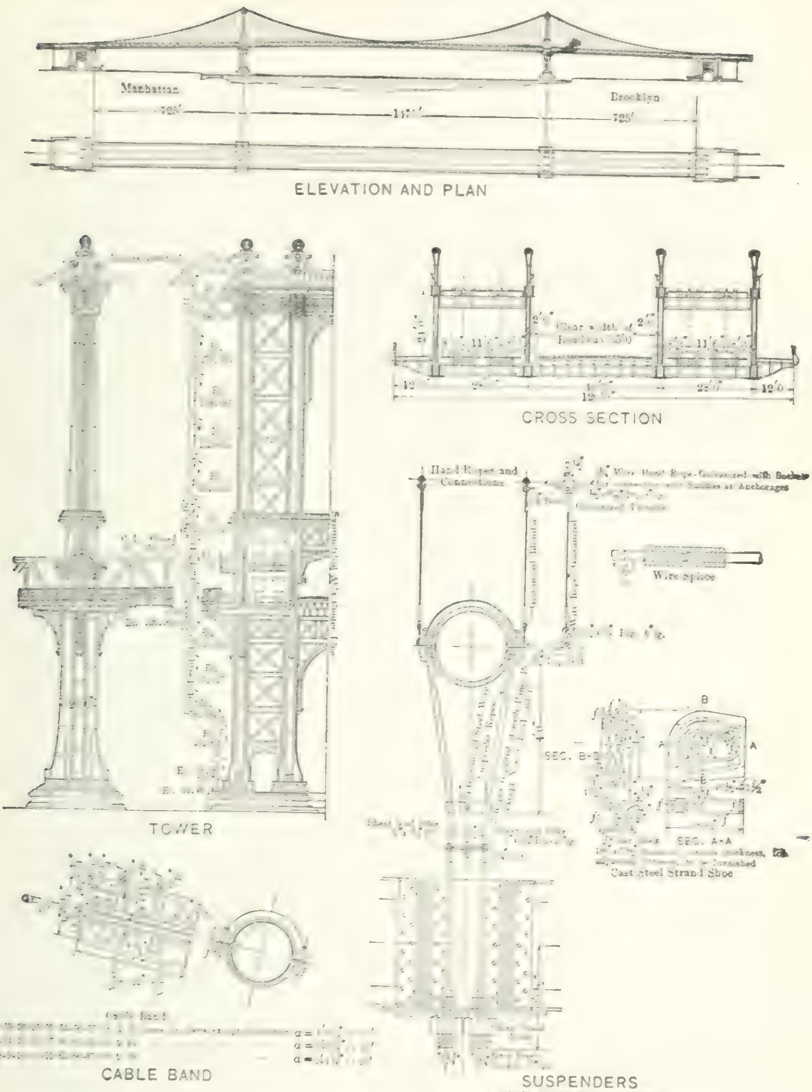


FIG. 36.—Manhattan Bridge.
(Type 2*S*).

rolled shapes (Figs. 29, 33, 34, 38 41). There may be one (Fig. 26), two (Fig. 30), or four suspenders (Figs. 32, 36) at a panel point.

If the hangers are made of rigid rods (instead of wire ropes), bending stresses due to lateral or longitudinal swaying of the bridge are avoided by inserting pin connections or links (Figs. 38, 39, 40).

Solid steel rods used for hangers generally have a high slenderness ratio and are subject to bending and to vibrations; to provide greater stiffness, tubular and built-up sections have been substituted (Figs. 34, 38, 41).

Where there are two or more suspenders at a panel point, the possibility of unequal division of load should be taken into consideration. Equalizers may be used to advantage (Fig. 32).

After passing around the cable band, the suspender may extend down as two separate ropes (Fig. 36); or the short end may be clamped to the main suspender, which then extends down as a single rope (Fig. 32).

The suspenders may connect directly to the floorbeams (Figs. 26, 27, 29, 38, 41), or to the top or bottom chord of the stiffening truss (Figs. 32, 36). In the latter case, the floorbeams frame into the chords or into the posts of the stiffening truss.

For connection to the floorbeams or chords, the suspenders may pass through and bear up against the lower cover plate with the aid of washers or special castings (Figs. 26, 32, 36); or they may loop around the floorbeams or chords either directly (Fig. 27) or with the aid of steel cross-pieces or yokes.

Connecting the suspenders to the top chord of the stiffening truss requires the entire length of the cable to be above the truss; this has aesthetic advantages (see Figs. 28, 34, 35), but it adds the depth of the truss to the required height of the towers. Lowering the cable saves height (Figs. 25–27, 29–33), but requires either lengthening of the floorbeams or spreading of the trusses or towers.

Another method of suspending the roadway is to loop the suspender rope under a small saddle casting from which there extend downward rigid rods terminating in holding nuts (Fig. 32) or steel flats bored to receive connecting pins.

Provision for adjustment of the hangers may be made by means of the holding nuts (Figs. 32, 30), or by means of sleeve nuts or turn buckles with right and left threads (such as shown in Fig. 32). Some engineers prefer to omit provision for adjustment, depending upon careful computation of required length before cutting the ropes and attaching the sockets.

18. Construction of Stiffening Trusses. The function of the stiffening truss is to limit the deformations of the cable and to so distribute any concentrated, unsymmetrical, or non-uniform loads as to keep the suspender tensions in a constant proportion (or equal if the cable is parabolic). In other words, the stiffening truss is required to hold the cable (or chain) in its initial curve of equilibrium. This will limit the deflections of the structure, and will resist the setting up of vertical oscillations.

The first suspension bridge provided with a stiffening truss was the 820-foot railway span at Niagara, built by John A. Roebling in 1851–55. The Brooklyn Bridge (Fig. 26), completed in 1883 by Roebling's son, was built with 4 cables and 6 stiffening trusses, and, in addition, was provided with diagonal stays.

Until comparatively recent years, stiffening trusses were only roughly figured and were made of constant section. The scientific design of suspension bridges dates from about 1898.

Stiffening trusses are generally built with parallel chords (Figs. 25, 30, 32, 35); a small variation in depth is sometimes introduced (Fig. 34). To prevent an unsightly and otherwise undesirable sag under load, and to counteract the illusion of sag, a generous camber is usually provided (Fig. 31).

The web system may be of the single Warren (Figs. 30, 35) double intersection (Fig. 34), or latticed types (Fig. 31). The *K*-truss may also be applied to advantage. Instead of a truss, plate girders may be used; the Vierendeel girder or quadrangular truss (like the floorbeam in Fig. 41) has also been proposed.

To make the design statically determinate, a hinge at the center of the stiffening truss is necessary (Type *3F* or *3S*, Fig. 26); but this construction has many drawbacks. In long spans, the angle change at the hinge would be so great as to cause serious bending stresses in the cable and overloading of adjacent

suspenders. Moreover, the wind stresses must be transferred through the hinge, and the details become more difficult and costly.

Making the truss continuous past the towers (Types $0F$, $0S$, Fig. 34) yields more effective stiffening: either less material is required or the deflections are reduced; furthermore, the impact effects of moving loads entering the main span are reduced. On the other hand, continuity renders the structure indeterminate (in the third degree); inaccuracies in construction, settlement of supports, and unequal warming of the chords will affect the stresses adversely.

Introducing hinges in the continuous stiffening truss relieves the indeterminateness and the accompanying uncertainty of stress conditions. In the Williamsburg Bridge (Fig. 31) a hinge is placed in each side span, close to the tower; in a prize design for the Elizabeth Bridge at Budapest (Fig. 34), two hinges were located in the main span; in both cases, the resulting system is singly indeterminate. In the usual two-hinged construction (Types $2F$, $2S$, Figs. 15, 36), the truss is hinged or interrupted at the towers.

As in other indeterminate structures, all precautions must be observed in construction to avoid false erection stresses. If the suspenders are adjustable, a definite apportionment of dead load between cable and stiffening truss may be secured. The stiffening truss may be totally relieved of dead-load stress by adjusting it under full dead load and mean temperature to the exact form it had when assembled in the shop at the same temperature; or by omitting certain members until full dead load (at mean temperature) is on the structure. In any case, the joints should not be riveted until the dead load is on and all adjustments are made.

The stiffening truss may be made of any height, depending upon the degree of stiffness desired. With increasing depth, the stiffness is naturally augmented; and, up to a certain limit, material is saved in the chords of the stiffening truss. Beyond this limit (economic depth = about $\frac{1}{40}$th of the span), the chord sections commence to increase as a result of the high temperature

stresses. For the sake of appearance, a somewhat shallower depth may be used (Fig. 35) without materially affecting the economy. If the truss has a center hinge, a greater depth becomes economical, since temperature stresses and uniform load stresses practically vanish. If the deformations are to be limited so as not to produce a deflection gradient exceeding 1 per cent, the depth must be made not less than $\frac{1}{75}$th of the span, whether two-hinged or three-hinged.

Bearings must be provided at the towers and abutments to take the positive reactions of the stiffening truss (even if continuous); and these points of the truss must also be anchored down to resist the uplift or negative reactions. At the expansion bearings, the anchorage must be so designed as to permit free horizontal movement; this may be accomplished either by the use of anchored rollers above the bearing, or by means of pin-connected rocker arms. One bearing of each truss should be fixed against horizontal movement, in order to resist longitudinal forces. (An exception was the Niagara Railway Suspension Bridge, where an automatic wedge device, for dividing the expansion equally between the two ends of the span, was provided.)

19. Braced-Chain Construction.—Overhead stiffening trusses may be regarded as inverted arches. A common form is the three-hinged truss with horizontal lower chord (Type $3BH$, Fig. 37), and these are designed similar to spandrel-braced arches. The chords and web members are built up of plates and angles with riveted panel points. The center hinge is designed to transmit the full value of H for dead and live load, and the maximum vertical shear from live load. At the towers, both chords are supported on expansion plates; the bottom chord ends there, but the top chord passes over cast-steel saddles and continues toward the anchorage. The top chord is supported at the top of the towers either on rockers or on rollers (Fig. 37).

In the three-hinged type ($3BH$), the center hinge may be located either in the upper or in the lower chord. In the former case, the upper chord will carry all of the dead load and full-span live load; the lower chord and web members will be

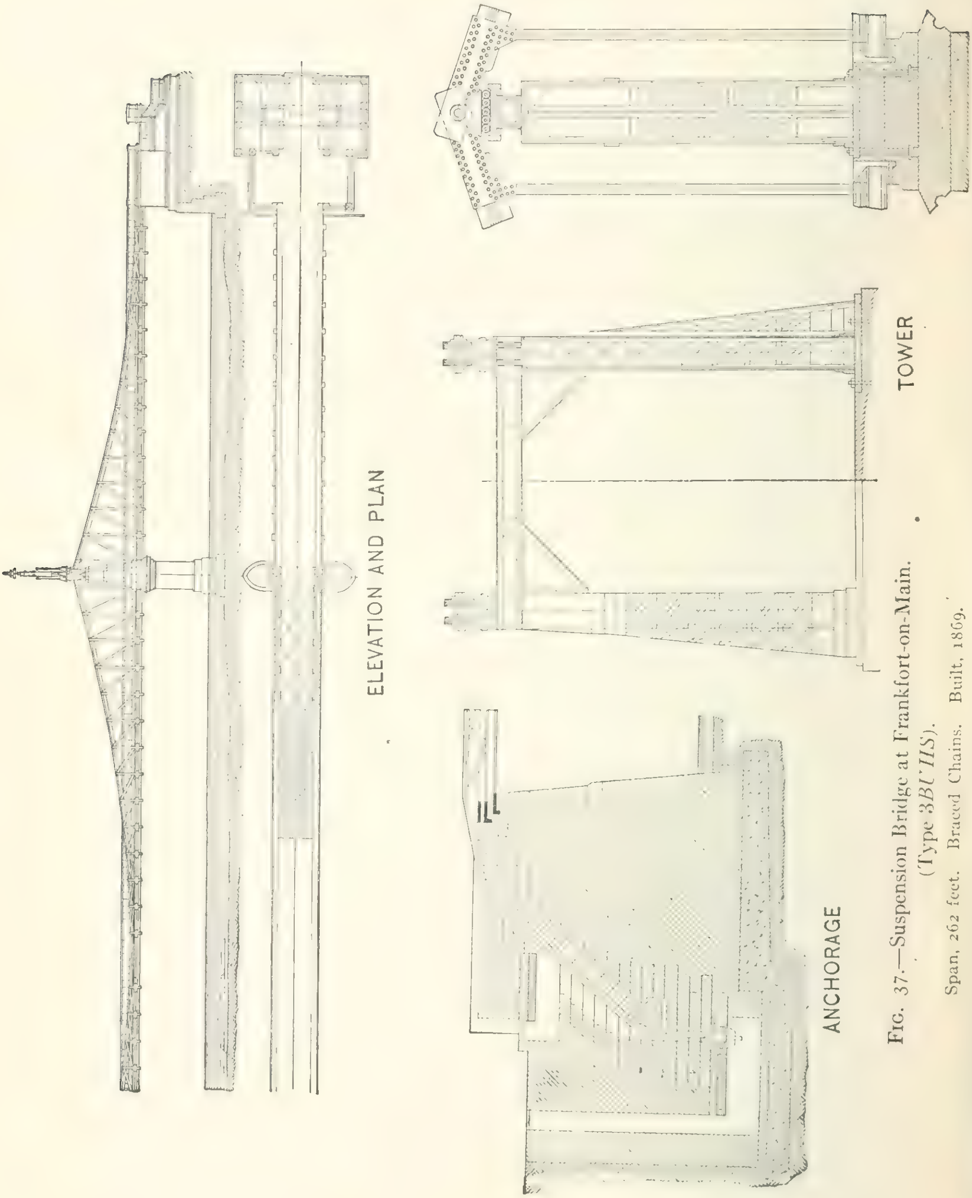


Fig. 37.—Suspension Bridge at Frankfort-on-Main.
(Type *3BUHS*).

Span, 262 feet. Braced Chains. Built, 1869.

stressed only by partial loading. Accordingly, the upper chord will have a fairly uniform section, while the other members will be comparatively light. This arrangement has a disadvantage, however, in the necessary break in the floor system under the hinge; the stringers must be provided with expansion joints, and the wind bracing must be interrupted.

If the hinge is placed in the lower chord, false members are required, for the sake of appearance, to cover the interruption in the top chord. Furthermore, there results a large variation in the chord stresses: near mid-span, the top chord stresses become light and the bottom chord stresses become heavy; and the reverse occurs near the towers.

The hinge may be either of the ordinary pin type or of the plate type. In the latter case, the chord section is concentrated into wide horizontal plates to connect the two halves of the span; and the vertical shears at the point are transmitted by means of vertical spring plates.

The false members in the interrupted chord may be connected with friction bolts in slotted holes, so that the resulting friction may act as a brake against oscillations.

To eliminate bending moments in the stiffening truss at the tower, the end member of the lower chord may be suspended from the saddle; or it may simply rest on an expansion bearing at the tower, with the end vertical omitted from the truss. Another arrangement is to make the tower integral with the main span truss, the tower being pivoted at the base, and the side span having only a hinged connection at the top of the tower.

The use of a wire cable for the top chord had an illustration in the Lambeth Bridge, London; but the details did not constitute an example worth copying. For long spans it is generally considered that the overhead bracing system cannot compete with the suspended stiffening truss, unless a wire cable is used. In any case, the use of a cable as a truss chord gives rise to difficulties in the detailing of connections at the panel points; and, despite noteworthy studies and designs (e.g., Fig. 39), the problems involved cannot be considered as fully solved.

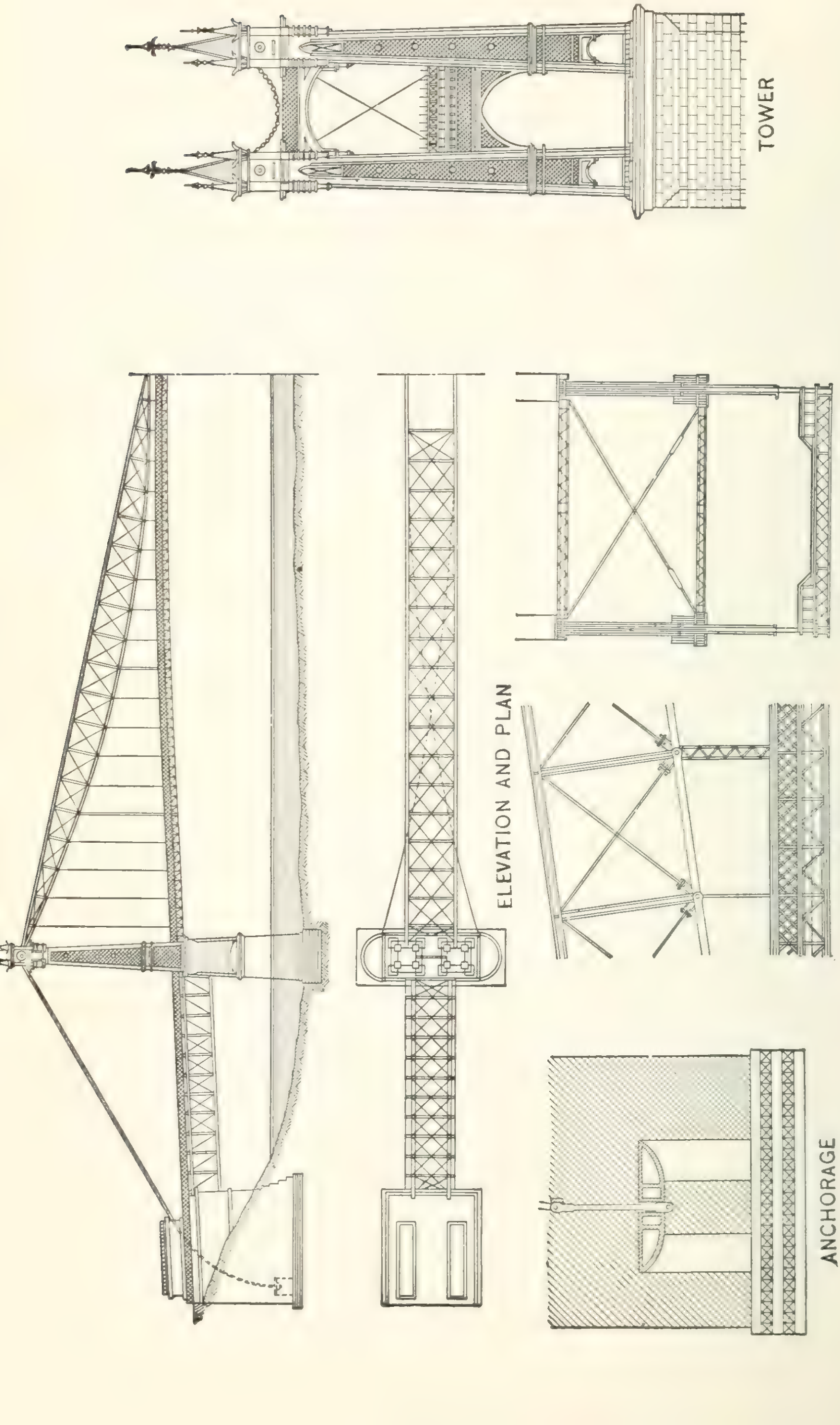


Fig. 38.—Point Bridge at Pittsburgh. (Type *3BLF*).
Span, 800 feet. Braced Chain. Erected 1877.

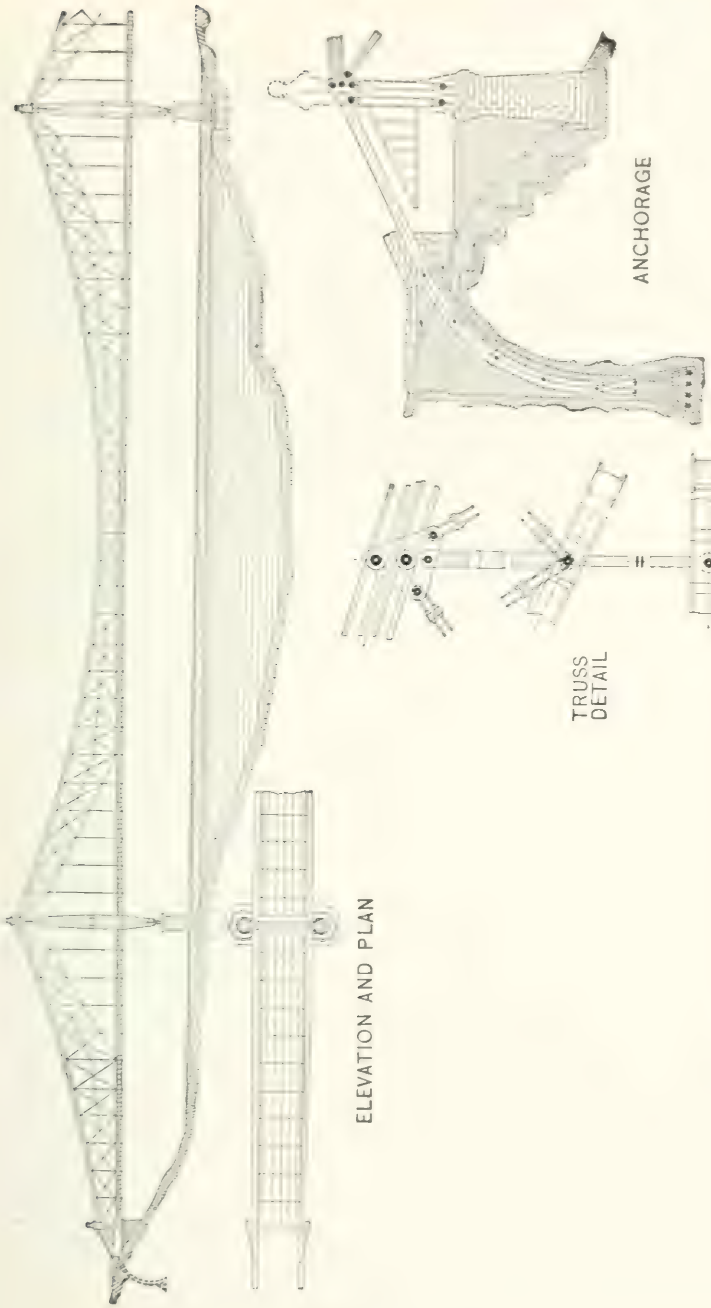


FIG. 39.—**Lindenthal's First Design for the Quebec Bridge. 1899. (Type 2*BVS*).**
Span, 1800 feet. Chain of pin-connected wire links.

When the braced-chain (or braced-cable) system is used, the web members should preferably not be connected until full dead load and half live load are on the structure at mean temperature. There will thus be accomplished a reduction in extreme stresses in the web members and lower chords; the maximum tension will be equal to the maximum compression in each member, and this stress will be only the arithmetic mean of the extreme stresses that would be produced without this precaution. However, if the design specifications prescribe stringent allowances for alternating stresses, the reduction in sections by this device will not be material.

The truss depth at the crown (Type *3B*) should preferably be between 0.15 and 0.3 of the sag of the chain. Sufficient depth must be provided to take care of the shearing stresses and to prevent undue flexibility in the central portion of the span; but excessive depth, besides increasing the metal required, impairs the desired graceful appearance of the suspension construction. If the hinge is omitted, any increase in crown-depth serves to augment the temperature stresses.

The form of braced-chain construction (Type *2BV*) proposed by Lindenthal for his first Quebec design (Fig. 39), and for the Manhattan Bridge, has many advantages. The bottom chord is bent up toward the towers, so as to obviate the necessity for long web members. The requirement of extra wind chords at the ends of the span is not, comparatively, an important objection.

Suspension trusses with bracing above the principal chain (Type *3BL*) are exemplified in the Point Bridge at Pittsburgh Fig. 38). In this system the stiffening chords are straight, and the bottom chord is parabolic. The top chord members have to resist both tension and compression.

A system having many advantages is the Fidler Truss (Type *3BC*), exemplified in Lindenthal's second Quebec design (Fig. 40), in a Tiber bridge at Rome, and in the Tower Bridge at London. In this system, consisting of two crescent-shaped half-arches, both chords are curved, the bottom chord having a sharper curvature than the equilibrium curve for full load.

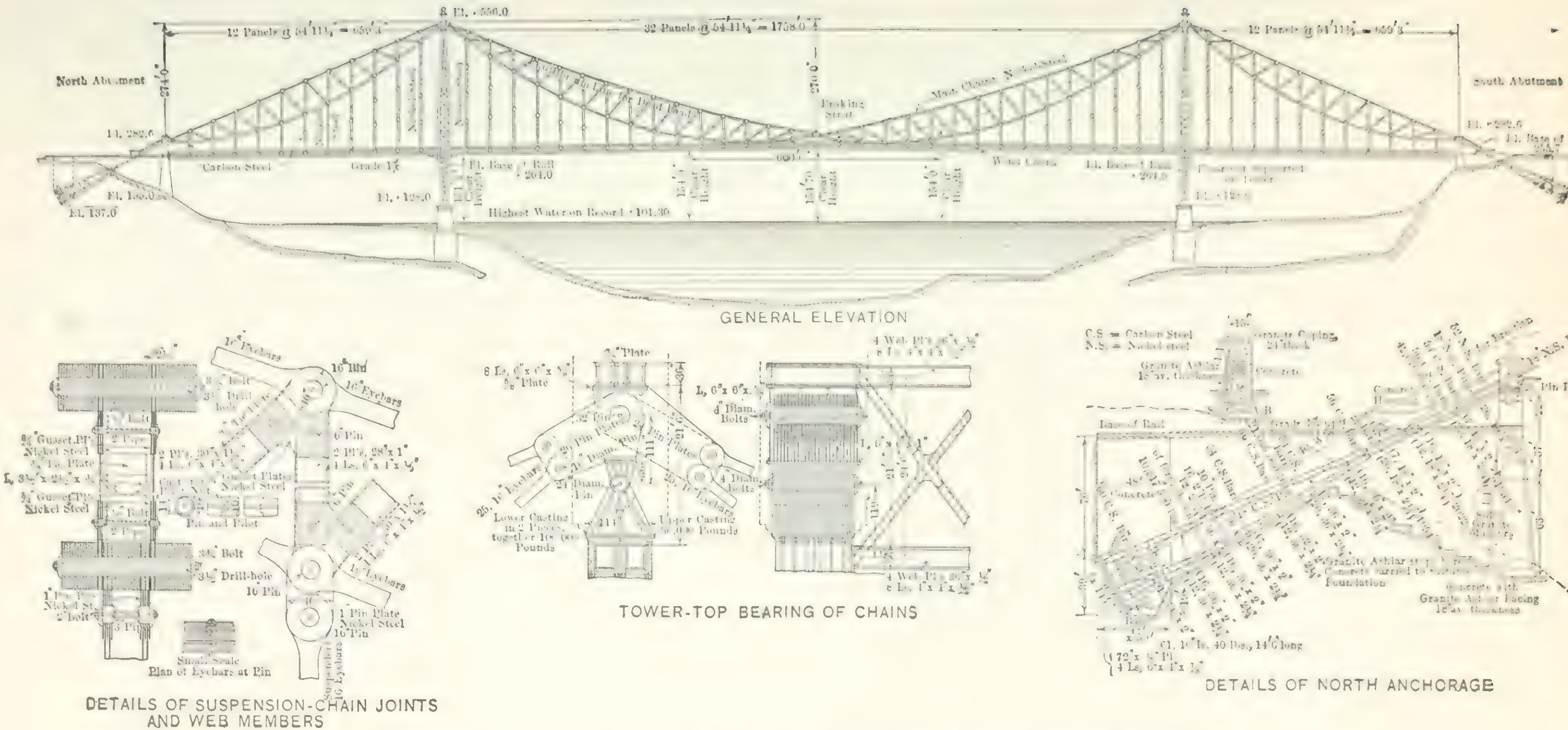


FIG. 40.—Lindenthal's Second Design for the Quebec Bridge (1910). (Type 3*BCS*)

Span, 1758 feet. Filler Truss.

The parabolic curve passes midway between the two chords, so that they are about equally stressed. With this system it is possible to avoid compressive stresses in both chords.

In the foregoing systems (Types $3BL$, $3BC$), it should be noted that the suspended floor system must be interrupted, or else of negligible moment of inertia, under the center hinge. It is more important here than in three-hinged arches, on account of the greater crown deflections in suspension systems.

The systems using parallel chains connected with web-bracing have been little used on account of the difficulties in stress analysis. If each chord has its own backstay (Type $0BP$), the system is threefold statically indeterminate. If the top chord is interrupted at the towers (Type $2BP$), the indeterminateness is reduced. It would be more effective, in such case, to bring the two chords together in crescent form instead of using parallel chords. In Lindenthal's Seventh St. Bridge at Pittsburgh and in his first design for the North River Bridge, the bottom chord rested directly on bearings on the top of the towers, and the top chord was connected thereto by a double quadrangular linkwork equivalent to a hinge; this made the system singly indeterminate (aside from the redundancy of web members). When parallel chains are used in the side spans, the bottom chord may be connected to the anchorage; or both chords may be brought together at a common pin for connection to the anchor chain.

The latest example of parallel chain construction is Lindenthal's new design for the Hudson River Bridge (Fig. 41). The main span is 3240 feet, and the bridge is continuous at the towers (Type $0BP$).

20. Wind and Sway Bracing.—To take care of transverse wind pressure and lateral forces from moving train loads, and to carry these forces to the piers, systems of lateral and sway bracing are required.

A system of wind bracing must be provided in the plane of the roadway, since the principal horizontal forces originate there. Such bracing system is obtained by inserting diagonals between the floorbeams, so as to form a horizontal truss; using

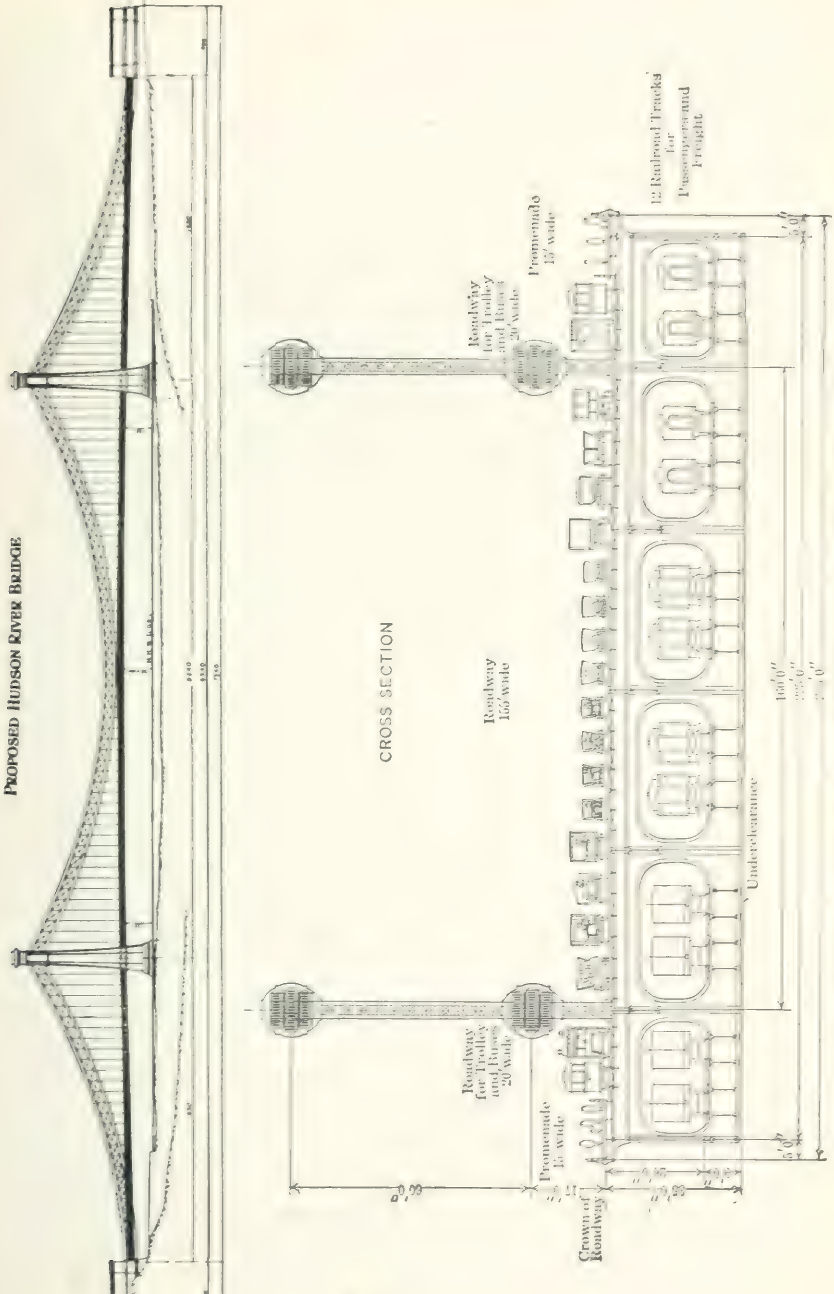


FIG. 41.—Proposed Hudson River Bridge. (Type *OBPS*).
Span, 3240 feet. Parallel Chains. Design by G. Lindenthal, 1921.

for chords either the bottom chords of the main stiffening truss (Figs. 26, 30, 32, 36, 37) or else adding extra longitudinals called wind chords (Figs. 38–41, 56).

If the stiffening truss is high enough to afford necessary clearances, a bracing system in the plane of the top chord may be added, giving a closed cross-section to the structure (Figs. 26, 32, 36, 43).

If the roadway is elevated, vertical sway frames of cross-bracing may be introduced between the trusses (Fig. 26); or the floorbeams may be built of deep latticed construction. In such case, a single plane of horizontal wind bracing will suffice.

A novel method of transverse bracing is introduced in the design for the Hudson River Bridge (Fig. 41) in the form of deep floorbeams (32 feet high) of quadrangular construction (Vierendeel Girders); the rectangular openings are used as passageways for the railway tracks.

Bearings must be provided at the towers to take the horizontal reactions of the wind truss without hindering longitudinal expansion. Vertical pins bearing in slotted guides may be used for this purpose.

The cables or chains present so small a surface to the wind as to require no wind bracing; or at most they may be connected together by horizontal ties.

On account of the inherent stable equilibrium of the suspension system, the wind bracing is to a certain extent relieved of its duty.

Braced-chain systems are provided with a single wind truss below the roadway, using either the lower chords of the main truss (Type *3BH*, Fig. 37) or special wind chords (Types *2BV*, *3BC*, etc., Figs. 38, 39, 40, 41). In addition, transverse sway frames are located at intervals between the two suspension trusses in the planes of verticals or diagonals, so far as clearance requirements permit (Fig. 38). In comparison with other types of bridges, but little material is needed for this sway bracing since, in the first place, the low position of the center of gravity makes the suspension truss stable without bracing, and, in the second place, there are no top chord compression members (in

most of these types) to be braced against buckling. It is essential, however, to provide properly designed rigid portal and sway bracing between the legs of the main towers (Figs. 37, 38).

A profusion of overhead bracing, besides being structurally unnecessary, will impair the graceful appearance sought in suspension constructions.

A center hinge in the stiffening truss introduces complications in the design of the wind-bracing system. If the hinge is, as usual, in the top chord, the wind bracing must follow the two central diagonals to make connection at the hinge; these central diagonals then act as wind-chord members, and their sections must be increased accordingly. If the top chord hinge lies above the roadway, the cross-bracing in the two central panels connecting with the hinge has to be omitted. This produces a point of weakness in the horizontal bracing system, and should be avoided in long spans either by omitting the hinge or by locating the hinge in the bottom chord.

Early suspension bridges that were found to have excessive lateral deflections and oscillations were stiffened by means of wind cables (wire ropes) placed in a horizontal plane under the roadway; these ropes were connected to the floorbeams and were anchored to the piers, so as to form a horizontal suspension system of cable and stiffening truss. To take care of wind in both directions, a double system of wind cables must be used, and their sag-ratio should be made as large as possible (Fig. 38). For greater stiffness, straight auxiliary cables have sometimes been added.

The efficacy of the above-described system of wind cables is doubtful, since it is ordinarily impossible to give the ropes proper initial tension; consequently they do not commence to take stress until the horizontal deflection of the structure exceeds a certain amount. For this reason, wind cables have not been relied upon for modern designs, but rigid wind trusses have been adopted instead, to take care of the wind pressures.

21. Towers.—For purposes of discussion, the tower may be considered as composed of two parts: the substructure or pier; and the tower proper, extending above the roadway and sup-

porting the cables or chains. The pier does not involve any special features differentiating it from ordinary bridge piers.

The tower must be designed so as not to obstruct the roadways. It is therefore composed of a column or tower leg for each suspension system (Figs. 30, 35–38). For lateral stability, the tower legs are braced together by means of cross-girders and cross-bracing (Figs. 30, 35) or by arched portals (Figs. 37, 38). The sway and portal bracing are necessary to brace the tower columns against buckling, to take care of lateral components from cradled cables or chains, and to carry wind stresses down to the piers.

The design of the tower depends upon the material employed. This is either masonry (Figs. 25, 27, 29, 33) or, more commonly steel (Figs. 15, 17, 28, 30–32, 34–41), and occasionally timber. If masonry is used, the tower may consist of shafts springing from a common base beneath the roadway and connected together at the top with gothic arches (Fig. 25); or, for smaller spans, the tower may consist of two separate tapering shafts or obelisks.

To meet architectural requirements and to express resistance to transverse forces, the outline of the tower should taper toward the top. This also conforms to structural requirements.

The tower legs must be designed as columns to withstand the vertical reaction of the cables; also as cantilevers to resist the unbalanced horizontal tension. The latter will depend upon the saddle design (fixed or movable), the temperature and loading conditions, and the difference in inclination of main and side-span cable tangents at the saddle. Forces due to wind pressure on the cables, towers, and trusses must also be provided for.

The application of steel to suspension-bridge towers offers many advantages. The lower cost permits a greater height in order to secure a more favorable sag-ratio. The thermal expansion of the steel tower balances that of the suspenders, so as to eliminate serious temperature stresses which would otherwise arise in indeterminate types ($2F$, $2S$, $0F$, $0S$).

Steel tower columns (Figs. 15, 17, 30, 36–39) are made up of plates and angles to form either open or closed cross-sections;

the sides may be either latticed or closed with cover plates. The relative dimensions are governed by the usual specifications for the design of compression members, particularly in respect to limiting unsupported widths of web plates. The cross-section enlarges toward the base, or outside stiffening webs are added; and the base must be anchored to resist the horizontal forces (Fig. 37).

For high towers, the individual legs may be made of braced-tower construction, each leg consisting of four columns spreading apart toward the base and connected with lacing or cross-bracing (Fig. 31).

Rocker towers, pin-bearing at the base, afford the most economical and scientific design for bridges of longer span. They eliminate the stresses from unbalanced horizontal forces without requiring movable saddle construction. The most notable examples actually constructed are the Elizabeth Bridge at Budapest (Fig. 34) and the Cologne Bridge (Fig. 17). (See also Figs. 15, 39, 40.)

If rocker towers are adopted, they must be secured against overturning during erection. This may be accomplished by temporary connections to the adjoining truss structure, by wedging or bracing at the base, or by guying the upper portions of the tower.

For foot bridges and bridges of small span, the simplest tower construction employs a stiffened plate for each leg, the two legs being braced together and carrying steel castings at the top to hold the cables.

If timber is used, each cable support consists of four battered posts with framed bracing, the two legs thus formed being connected at the top with cross-bracing.

22. Saddles and Knuckles.—The cables are generally continuous over saddles on top of the towers.

Designs have been made with the main cables terminating at the towers, with a special connection at the top of the towers to the backstay cables (e.g., Morison's North River design). The advantages claimed were shorter cable strands to handle in erection, elimination of stresses due to bending of the cable

over the towers, and the possibility of increasing the section of the backstays to permit steeper inclination. The latter advantages, however, can be secured with continuous cables by employing certain design features.

If the stress in the backstay, as a result of steeper inclination, is much greater than in the main cables, auxiliary strands may be incorporated in the backstay to increase its section; and provision should be made for the connection of these auxiliary strands to the saddle. (An example is the Rondout Bridge at Kingston, N. Y., Fig. 56.)

Cable saddles are generally made of cast steel (Fig. 32). They are either bolted to the top of the tower (Fig. 36) or provided with rollers (Figs. 32, 33, 37).

Where fixed saddles are used (Fig. 36), the resultant unbalanced horizontal forces must be calculated and provided for in the design of the tower, unless the tower is made of rocker type (Figs. 15, 17, 34, 39, 40). If the saddles are movable, the eccentricity of the vertical reaction under various loading conditions must be provided for.

The simplest but least satisfactory construction, used in some smaller bridges, consists of fixed saddles over which the chain or cable is permitted to slide. In early cable bridges, the wrapping was discontinued near the towers, and the wires were spread out to a flat section to pass over the saddle casting; this is objectionable as it gives access to moisture. It is preferable to give the saddle a cross-section conforming to the cable section; to reduce wear from the rubbing, the cable may be protected by a lead sleeve. On account of the friction, this arrangement does not eliminate the unbalanced horizontal pull on the top of the tower. On the whole, this construction, or any sliding saddle arrangement, is not to be recommended.

Another saddle arrangement consists of steel pulleys, free to rotate, over which the cable passes (Fig. 27). A similar arrangement used with chains consists of a fixed roller nest over which curved eyebars slide (Fig. 33); the resulting bending stresses, however, are objectionable.

The best arrangement to permit horizontal movement on

top of the tower consists of a roller support for the saddle (Figs. 32, 37). In modern designs, the rollers are of equal height between two plane surfaces. The construction comprises a bed plate, a nest of rollers connected by distance bars, and the superimposed saddle casting. In the saddle rests the cable (Fig. 32) which is held from sliding by friction or clamping. Instead of a saddle, the movable part may consist of a casting to which the chains are pin-connected (Fig. 37). The resultant of the tensions of the cables or chains should pass through the middle of the roller nest to give an even distribution of stress. The friction of the rollers is so small that the obliquity of the resultant reaction is negligible.

Instead of circular rollers, segmental rollers (rockers) may be used, so as to furnish a greater diameter and thereby reduce friction and roller-bearing stress. Segmental rollers, however, must be secured against excessive motion liable to cause overturning.

Rollers (Figs. 32, 33, 37) serve to reduce the bending stresses on the towers due to unbalanced horizontal cable pull resulting from temperature and special loading conditions. On the other hand, they add expense, increase erection complications, give trouble in maintenance, and merely substitute eccentric vertical loading for unbalanced horizontal pull. On the whole, fixed saddles provide a simpler and safer construction.

Another saddle design consists of a rocker, pin-connected at either upper or lower end to the tower and carrying the cable or chain at the other end. The rocker-hanger or pendulum type has been used only in earlier bridges; the main objection to it is the increased height of tower required. The rocker-post type (Fig. 39) has pin-connection to cable or chain at the upper end, and has a pin or cylindrical bearing at the lower end.

Short rocker posts should not be used for long spans; after such posts assume an inclined position under temperature variation, the return to normal position is seriously resisted by the necessity of raising the point of cable support through a vertical height.

The tower itself may be made to serve as a rocker post for its

full height by providing hinge action at the base. This construction was used in 1857 for a bridge over the Aare at Berne; also in the Elizabeth Bridge at Budapest, 1903 (Fig. 34), in Lindenthal's designs for the Quebec Bridge, 1899, 1910 (Figs. 39, 40) and for the Manhattan Bridge, 1902; in the Rhine Bridge at Cologne, 1915 (Fig. 17), and in the design for the Detroit Bridge, 1921 (Fig. 15). Instead of using pins at the lower end, the hinge-action may be secured by providing the tower leg with a segmental base (Fig. 17) or with a concave nest of rollers (Fig. 15).

Knuckles are provided in the anchorages at all points where the backstays or anchor chains alter their direction (Figs. 29, 32, 33, 36 40, 42). They are similar in function to tower saddles, and should be designed to permit any movement due to thermal or elastic elongation of the anchor chain.

Sliding bearings (Figs. 37, 39) are commonly used at knuckle joints, and may be considered suitable where the directional change is small. In the Rondout Bridge at Kingston, N. Y. (Fig. 56), the design consists of vertical pin plates supporting the knuckle pin and sliding on steel plates anchored in the masonry.

Roller bearings (Figs. 33, 36, 40) give a better design for the anchorage knuckles. A cable may be carried in a saddle casting resting on rollers (Fig. 36); and chain eyebars may be either directly supported on rollers (Fig. 33) or may be pin-connected to a casting carried on rollers (Fig. 40). The plane of the rollers should be normal to the bisector of the angle of the chain or cable.

Rocker supports (Figs. 32, 39, 42) are also used for anchorage knuckles. The change in direction may be accomplished at one main rocker strut (Fig. 42), or may be distributed over a large number of small rocker knuckles (Fig. 32). The direction of the rocker should preferably coincide with the bisector of the angle of the chain or cable.

23. Anchorages.—The safety of a suspension bridge depends upon the security of the anchorages. Consequently, in any new design, the anchorages should receive thorough study and their construction should be carefully supervised; and, after

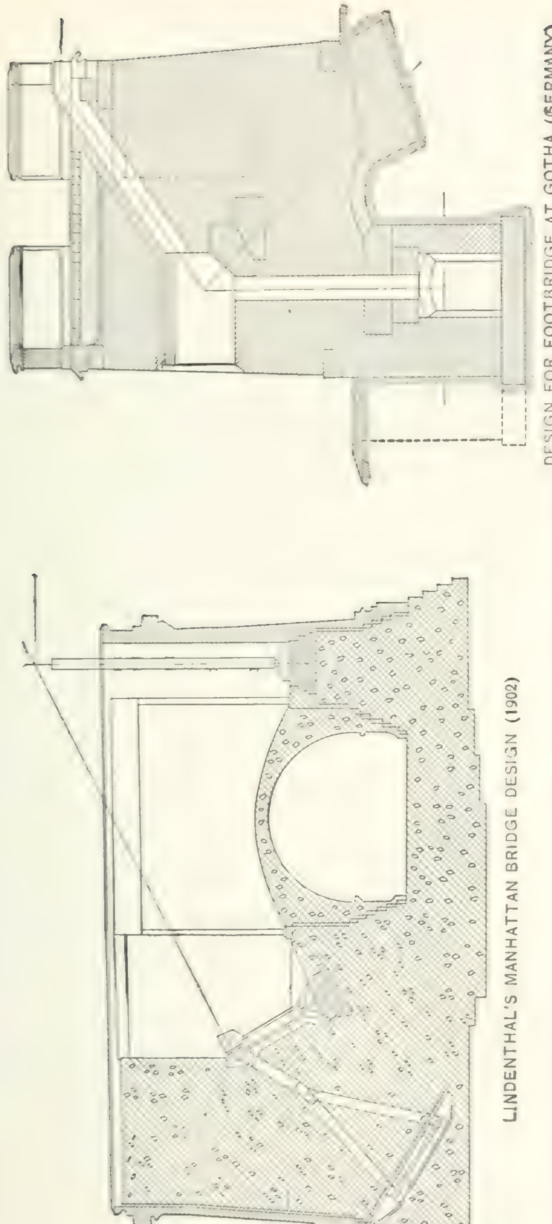


FIG. 42.—Designs for Anchorages.

completion, the condition of the anchorage should receive watchful attention. Accessibility for inspection and maintenance should be considered in the design.

In rare cases it is possible to anchor the cables in natural rock (Figs. 27, 39, 40). The shaft or tunnel which is to contain the anchor chain must then be driven to such depth as to reach and penetrate rock that is perfectly sound, proof against weathering and of sufficient thickness to afford the necessary anchorage.

In most cases, the anchorage requires a masonry construction which resists the cable pull by friction on its base or by the resisting pressure of the abutting earth (Fig. 34).

Masonry anchorages may be imbedded below ground level, with backstays connecting them to the nearest towers (Fig. 29); or they may constitute the end abutments of the side spans (Figs. 26, 28, 30, 32–38, 41). The latter arrangement generally requires bending or curving the line of the cable or chain from its initial inclination to a more vertical direction, in order to secure the necessary depth of anchor plate without excessive length of anchorage (Figs. 32, 33, 36–38, 42). In addition to stability against sliding, such anchorage must also be designed for stability against tilting or overturning. Furthermore, the applied forces must be followed through the masonry and the resulting normal and shearing stresses at all sections and joints must be provided for. The extreme pressures on the base should also be investigated, to make sure that they do not exceed the allowable load on the foundation material (Fig. 44). Foundations on piles (Figs. 32, 36) should be avoided, as they give insufficient security against displacement of the anchorage; if such foundations are unavoidable, an ample proportion of batter piles should be provided.

The anchor chains go through the masonry and are fastened at their ends to anchor plates or to reaction girders. In larger structures, anchor tunnels may be left in the masonry, affording access for inspection of the anchor chains (Figs. 33, 34, 37, 42).

As a rule, each cable or chain is separately anchored; in rare cases, the two cables have been connected in the anchorage so as to form a loop around a body of masonry or rock.

The anchor chains are commonly secured by means of an anchor plate which is either a single casting or built up of cast sections; the last link is passed through this anchor plate and is fastened behind it with a special pin or bolt (Figs. 33, 37, 38). The anchor plate is stiffened against bending by means of perpendicular webs or ribs (Figs. 33, 38); it must have a bearing surface large enough to transfer and distribute the pressure to a sufficient area and mass of masonry.

Instead of cast-steel anchor plates, anchor girders built up of rolled sections have been used in more recent designs (Figs. 32, 39, 40, 42). The chains are pin-connected to the webs of these girders, and the latter transmit the reaction to grillages or castings bearing against the masonry.

Provision for adjustment of backstay length may be made in the anchorage, if not elsewhere. Adjustment may be secured through the use of wedges in the connection behind the anchor plate (Fig. 37); or by means of a threaded connection between the strand sockets and round rods passing through the anchor plate.

The anchor plates (Figs. 33, 37, 38) are designed like any other masonry plates. The area is determined by the allowable bearing pressure on the concrete or stone, and the section of the plate and stiffening ribs are determined by the shearing and bending stresses. The holding bolts or wedges must also be proportioned for shearing and bending stresses; for greater bending strength, the bolts may be made of oval rather than circular section.

A connection of cable to anchor chain is illustrated in Fig. 32.

In small bridges, the cable is often anchored directly, by passing the wire slings around anchored bolts or around anchor blocks. This method can be used only with parallel wire strands; and, if the diameter of the sling is too small, excessive bending stresses will arise in the wire.

At each change in direction of the cable or chain in the anchorage, a bearing is required. This may consist of a casting with rounded surface over which the cable or chain may slide

(Figs. 37, 39), or of a flat plate on which the eyebar heads rest. A knuckle casting with bearing for the eyebar pin is preferable to one on which the eyebar heads bear. If the change in direction of the anchor chain is considerable, roller bearings (Figs. 33, 36, 40) or rockers (Figs. 32, 42) must be provided.

In general, aside from greater simplicity, a straight anchor chain (Figs. 30, 34) is preferable to a curved or bent chain (Figs. 29, 32, 33, 36–38, 42), as the latter results in greater lengthening of the cable from compression or settlement of the masonry. Space limitations, however, frequently make this arrangement unavoidable.

If it is desired to leave the anchorage steel accessible, shafts or tunnels must be provided in the masonry, large enough for a man to pass through (Figs. 27, 29, 33, 34, 37, 42); a clearance of 2 to 3 feet is necessary. Near the lower end, the shaft generally becomes constricted in order to reduce the required size of the anchor plate; consequently the end of the chain is not fully accessible to inspection. For the examination of the anchor plate and fastenings, vaulted chambers or horizontal passageways (about 3 to 4 feet wide and 5 to 6 feet high) are provided behind the anchor plate (Figs. 33, 37, 38, 42). These chambers may lead to the sides of the anchorage, where they are sealed by doors (Figs. 33, 42), or they may be reached through horizontal tunnels (Fig. 27). Inclined shafts (Figs. 27, 33, 37, 42) may be roofed with stepped slabs, flat slabs or an arched vault. Rain and dirt must be excluded, and the points of emergence of the cables or chains should be protected accordingly.

Instead of leaving open passageways for inspection and painting, the opposite course may be adopted and the anchorage completely sealed against the entrance of air or water (Figs. 39, 40, 42). In such designs, the anchorage steel is imbedded in concrete, or surrounded with waterproofing material, so as to exclude air and moisture and thereby prevent oxidation. The shafts receiving the anchor chains or cables are made as narrow as possible, and are subsequently filled with concrete, cement mortar, asphaltic cement or other waterproofing substance.

The anchorage may be built of stone masonry or of concrete.

The use of reinforced concrete gives maximum flexibility in design.

The forces acting on the anchorage as a whole are the cable pull, the weight of the masonry and any superimposed reactions, and the pressure or resistance of abutting earth. For the study of the internal stresses, the outside cable pull is replaced by the reactions of the anchor plates and the knuckle castings. By graphic composition of these various applied forces, the lines of pressure in the masonry are determined. The resultant of all the external forces, including the weight of the anchorage, must intersect the base within the limits necessary to prevent uplift at the heel (Fig. 44); and the inclination of the resultant from the normal must not exceed the angle of friction. If it proves impracticable to secure this stability against sliding with a horizontal foundation, the base may be sloped (Fig. 42) or stepped to increase the sliding resistance. Stepping the base is not effective save on hard foundations. In soft ground, requiring piles, the pile caps should be imbedded in masonry; and the piles should preferably be battered in the direction of the resultant pressure.

Granite or other stone blocks should preferably be used to take the direct pressure of the anchor plate and knuckle castings (Figs. 32, 33, 37, 42). Extending forward from the anchor plate, cut-stone blocks may be laid in arch formation, following the curving line of resultant pressure and with joints normal thereto. The rest of the anchorage, serving only to provide weight, may be built of rubble masonry or brickwork in horizontal courses; or of rubble concrete.

Special designs of anchorages are frequently necessitated by physical conditions and economic limitations. In one design for a South American bridge, the author devised a buttressed concrete box filled with sand to give it weight. In another design, he used a concrete tower encasing the anchor chain and supporting its saddle, the tower being braced by a flying buttress of concrete delivering the resultant to an inclined foundation. For the design of the Detroit-Windsor Bridge (Fig. 15), on account of the depth to rock, there was devised by C. E.

Fowler and the author an articulated type of anchorage consisting of two members: an anchor chain in an inclined shaft leading into rock, and a heavy steel strut in an oppositely inclined shaft carrying the resultant thrust to rock, both shafts being filled with waterproof concrete for the protection of the steel; the two members form an inverted V, the cable being attached at their junction.

CHAPTER III

TYPICAL DESIGN COMPUTATIONS

(NOTE.—All references are to Chapter I, "Stresses in Suspension Bridges")

EXAMPLE 1

Calculations for Two-hinged Suspension Bridge with Straight Backstays (Type 2F)

1. Dimensions.—The following dimensions are given:

l = Main Span = 50 panels at 22.5 ft. = 1125 ft. $(l' = l)$.

l_2 = Side Span = 281.25 ft. $= \frac{l}{4}$.

f = Cable-sag in main span = 112.5 ft. $\left(n = \frac{f}{l} = \frac{1}{10}\right)$.

f_1 = Cable-sag in side spans = 0. (Straight backstays).

d = Depth of Truss = 22.5 ft.

Mean Chord Sections (gross):

Top = 94 sq. in. Bottom = 161 sq. in.

I = Mean Moment of Inertia of stiffening truss in main span $= 94(14.2)^2 + 161(8.3)^2 = 30{,}000$ in.^{2} ft.^{2} (1 truss).

Width, center to center of trusses or cables = 34.5 ft.

A = Cable Section = 84 sq. in per cable. $(A_1 = A)$.

$\tan\alpha$ = Slope of Cable Chord in main span = 0.

$\tan\alpha_1$ = Slope of Cable Chord in side span $= 4\frac{f}{l} = 4n = 0.4$.

2. Stresses in Cable.—Given:

w = Dead Load per cable (including cable)
= 2650 lb. per lin. ft.

p' = Live Load per cable = 850 lb. per lin. ft.
t = Assumed Temperature Variation = $\pm 60°$ F.
($E\omega t = 11{,}720$ lb. per sq. in.)

All values are given per cable.

For Dead Load, by Eq. (11), the horizontal component of cable stress is,

$$H = \frac{wl^2}{8f} = \frac{10}{8} wl = 3730 \text{ kips.} \quad (1 \text{ kip} = 1000 \text{ lb.})$$

For Live Load, by Eq. (167), the denominator of the formula for H is,

$$N = \frac{8}{5} + \frac{3I}{Af^2} \cdot \frac{l'}{l} \cdot (1 + 8n^2) + \frac{6I}{A_1 f^2} \cdot \frac{l_2}{l} \cdot \sec^3 \alpha_1$$
$$= \tfrac{8}{5} + 3(.0283)(1 + .08) + 6(.0283)\tfrac{1}{4}(1.08)^3 = 1.745.$$

By Eq. (168), the live-load tension will be,

$$H = \frac{p'l}{5Nn} = \frac{2}{N} \cdot p'l = 1.146\, p'l = 1095 \text{ kips.}$$

The total length of cable between anchorages is given by Eq. (176):

$$\frac{L}{l} = (1 + \tfrac{8}{3}n^2) + 2 \cdot \frac{l_2}{l} \cdot \sec \alpha_1 = (1 + .027) + \tfrac{2}{4}(1.08) = 1.567.$$

Then, for temperature, by Eq. (156),

$$H_t = -\frac{3EI\omega tL}{f^2 Nl} = \mp \frac{3(30{,}000)}{(112.5)^2} \cdot \frac{1.567}{1.745}(11{,}720) = \mp 75 \text{ kips.}$$

Adding the values found for H:

D. L.	3730 kips
L. L.	1095
Temp.	75

we obtain, Total $H = 4900$ kips per cable.

The maximum tension in the cable is, by Eq. (5),

$$T_1 = H \cdot \sec \alpha_1 = H(1.08) = 5300 \text{ kips.}$$

At 65,000 lb. per sq. in., the cable section required is $5300 \div 65 = 82$ sq. in. (Section provided = 84 sq. in.)

3. Moments in Stiffening Truss.—Given:

Live Load $= p = 1600$ lb. per lin. ft. per truss.

(All values given and calculated are per truss.)

With the main span completely loaded, the bending moment at any section x is given by Eq. (169):

$$\text{Total } M = \tfrac{1}{2}px(l-x)\left(1 - \frac{8}{5N}\right) = \tfrac{1}{2}px(l-x)(.085).$$

In other words, only 8.5 per cent of the full-span live load is carried by the truss. Accordingly, at the center,

$$\text{Total } M = .085\frac{pl^2}{8} = +21{,}500 \text{ ft. kips per truss.}$$

At other points, the values of M are proportional to the ordinates of a parabola. They are obtained as follows:

Section $\left(\frac{x}{l}\right)$	Parabolic Coefficient	Total M (ft. kips)
0	$4 \times 0 \times 1 = 0$	0
0.1	$4 \times .1 \times .9 = .36$	+ 7,800
0.2	$4 \times .2 \times .8 = .64$	+13,800
0.3	$4 \times .3 \times .7 = .84$	+18,100
0.4	$4 \times .4 \times .6 = .96$	+20,600
0.45	$4 \times .45 \times .55 = .99$	+21,300
0.5	$4 \times .5 \times .5 = 1.00$	+21,500

For maximum and minimum moments, the critical points are found by solving Eq. (142):

$$C(k) = N \cdot n \cdot \frac{x}{y} = .1745\frac{x}{y}.$$

The values of the minimum moments are then given by Eq. (170):

$$\text{Min. } M = -\frac{2px(l-x)}{5N} \cdot D(k) = -\frac{4}{5N-8} \cdot (\text{Total } M) \cdot D(k)$$
$$= -5.52(\text{Total } M) \cdot D(k).$$

The tabulations in Table I or the graphs in Fig. 12 are used in solving the values of $C(k)$ for k and then finding the correspond-

ing values of $D(k)$. The following tabulation shows the successive steps:

$\frac{x}{l}$	$\frac{y}{l}$	$\frac{x}{y}$	$C(k)$	k	$D(k)$	Min. M (ft. kips)
0	0	(2.50)	(.436)	(.355)	(.531)	0
.1	.036	2.78	.485	.392	.437	−18,800
.2	.064	3.12	.544	.437	.334	−25,400
.3	.084	3.57	.623	.497	.224	−22,400
.4	.096	4.17	.728	.584	.113	−12,800
.45	.099	4.55	.795	.647	.061+.001	− 7,300
.5	.100	5.00	.872	.729	.024+.024	− 5,700
.55	.099	5.55	.970	.876	.001+.061	− 7,300

(By using Fig. 13, $D(k)$ may be found directly from the values of $C(k)$.)

For all sections between $x=\frac{N}{4}\cdot l=.436l$ and the symmetrical point $x=.564l$, a correction is made in the above table for the second critical point, as explained under Eq. (143′).

To find the maximum moments, apply Eq. (144):

$$\text{Max. } M = \text{Total } M - \text{Min. } M.$$

Dividing the maximum and minimum moments by the truss depth ($d=22.5$ ft.), we obtain the respective chord stresses as follows:

Section $\frac{x}{l}$	Maximum M (ft. kips)	Chord Stresses (kips)	
		Maximum	Minimum
0	0	0	0
0.1	+26,600	∓1180	± 840
0.2	+39,200	∓1740	±1130
0.3	+40,500	∓1800	±1000
0.4	+33,400	∓1490	± 570
0.45	+28,600	∓1270	± 320
0.5	+27,200	∓1210	± 250

In this tabulation of the stresses, the upper signs refer to the top chord and the lower signs to the bottom chord. Dividing the above values by the specified unit stresses in tension and compression, respectively, we obtain the required net and gross sections of the chord members. For the bottom chords, these sections must be increased to provide for the wind stresses, computed as indicated below. In addition, the temperature stresses must be taken into consideration.

The moments produced by temperature variation are given by Eq. (157):

$$M_t = -H_t \cdot y.$$

As found above, $H_t = \mp 75$ kips for a temperature variation of $\pm 60°$ F. At the center,

$$M_t = \pm(75 \text{ kips} \times 112.5 \text{ ft.}) = \pm 8450 \text{ ft. kips.}$$

At other sections, the moments are proportional to the parabolic ordinates y:

Section	Parabolic Coefficient	M_t
$\frac{x}{l}=0$	0	0
.1	.36	± 3040
.2	.64	± 5400
.3	.84	± 7100
.4	.96	± 8100
.5	1.00	± 8450

(The upper signs pertain to a rise in temperature, the lower signs to a fall.)

The resulting stresses are to be combined with the live-load stresses previously found, in whatever manner the specifications may prescribe. For the sections $\frac{x}{l} = 0$ to 0.4, the temperature moments amount to less than 25 per cent of the live-load Max. M, in which case, according to some specifications, the temperature stresses may be ignored.

4. Shears in Stiffening Truss.—

($p = 1600$ lb. per lin. ft., $\frac{1}{2}pl = 900$ kips.)

With the main span fully loaded, the shears at the various sections are given by Eq. (173):

$$\text{Total } V = \tfrac{1}{2}p(l-2x)\left(1-\frac{8}{5N}\right) = \tfrac{1}{2}pl\left(1-\frac{2x}{l}\right)(.085) = 76.5\left(1-\frac{2x}{l}\right).$$

Section	$\left(1-\frac{2x}{l}\right)$	Total V
$\frac{x}{l}=0$	1	76 kips
.1	.8	61
.2	.6	46
.3	.4	31
.4	.2	15
.5	0	0

The maximum shears are given by Eq. (149):

$$\text{Max. } V = \tfrac{1}{2}pl\left(1-\frac{x}{l}\right)^2\left[1-\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G\left(\frac{x}{l}\right)\right].$$

The values of $G\left(\frac{x}{l}\right)$ are taken from Table I, and the shears are obtained as follows:

Section	$\left(1-\frac{x}{l}\right)^2$	$\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)$	$G\left(\frac{x}{l}\right)$	[ ]	Maximum V
$\frac{x}{l}=0$	1	2.29	.400	.084	+ 76+219
.1	.81	1.84	.482	.114	+ 83+110
.2	.64	1.38	.565	.220	+131+ 28
.3	.49	0.92	.647	.405	+179
.4	.36	0.46	.726	.666	+216
.5	.25	0	.800	1.000	+225 kips

For all sections $x < \frac{l}{2}\left(1 - \frac{N}{4}\right) = .282l$, the loading for maximum shear extends from the given section x to a critical point kl defined by Eq. (150):

$$C(k) = \frac{N}{4} \cdot \frac{l}{l - 2x} = \frac{.436}{1 - \frac{2x}{l}}.$$

The values $C(k)$ are solved for k with the aid of Fig. 13.

Section	$1 - \frac{2x}{l}$	$C(k)$	k
0	1	.436	.355
.1	.8	.545	.438
.2	.6	.730	.588

For these sections, a correction is to be added to the values of Max. V found above. This additional shear is given by Eq. (151):

$$\text{Add. } V = \tfrac{1}{2}pl(1-k)^2\left[\frac{8}{N}\left(\frac{1}{2} - \frac{x}{l}\right) \cdot G(k) - 1\right].$$

Section	k	$(1-k)^2$	$\frac{8}{N}\left(\frac{1}{2} - \frac{x}{l}\right)$	$G(k)$	$[\ ]$	Add. V
$\frac{x}{l} = 0$	.355	.416	2.29	.691	.585	+219 kips
.1	.438	.316	1.84	.755	.389	+110
.2	.588	.170	1.38	.858	.185	+ 28

(By using Fig. 13, $G(k)$ may be found directly from the values of $C(k)$.)

The minimum shears are then given by Eq. (153):

$$\text{Min. } V = \text{Total } V - \text{Max. } V.$$

Section	Total V	Maximum V	Minimum V.
$\frac{x}{l}=0$	+76 kips	+295 kips	−219 kips
.1	+61	+193	−132
.2	+46	+159	−113
.3	+31	+179	−148
.4	+15	+216	−201
.5	0	+225	−225

The temperature shears are given by Eq. (158):

$$V_t = -H_t \cdot (\tan\phi - \tan\alpha).$$

In this case, $\tan\alpha = 0$, and $H_t = \mp 75$ kips. At the ends,

$$\tan\phi = \frac{4f}{l} = 4n = 0.40,$$

and the slope $(\tan\phi)$ diminishes uniformly toward the crown.

Section	$\tan\phi$	V_t
$\frac{x}{l}=0$	0.40	±30 kips
.1	.32	±24
.2	.24	±18
.3	.16	±12
.4	.08	± 6
.5	0	0

These shears are to be combined with the maximum and minimum live-load shears, as may be required by the specifications.

5. Wind Stresses in Bottom Chords.—

(Assumed wind load $= p = 400$ lb. per lin. ft.)

If the lateral bracing is in the plane of the bottom chords, these chords act as the chords of a wind truss. The applied wind pressure p is partly counteracted by a force of restitution r due to the horizontal displacement of the weight of the stiffening

truss w. The resulting reduction in the effective horizontal load is given with sufficient accuracy by the formula,

$$\frac{r}{p}=\frac{.013\dfrac{wl^4}{vEI}}{1+.013\dfrac{wl^4}{vEI}}.$$

(For the derivation of this formula, see Steinman's "Suspension Bridges and Cantilevers," D. Van Nostrand Co., 1913, page 76.) In this case, w = total dead load (both trusses) = 5300 lb. per lin. ft.; v = vertical height from cable chord to center of gravity of the dead load = 135 ft.; I = moment of inertia of wind truss = $\frac{1}{2}(161)\times(34.5)^2 = 96{,}000$ in.^{2}ft.^{2} Substituting these values, we obtain,

$$\frac{r}{p}=\frac{.284}{1+.284}=0.22.$$

Hence the force of restitution r (due to the obliquity of suspension after horizontal deflection) amounts, in this case, to 22 per cent of the applied wind load (p) at the center of the span. The force r diminishes to zero at the ends of the span, and the equivalent uniform value of r may be taken as $\frac{5}{6}$ of the mid-span value. The resultant horizontal load on the span is,

$$p-\tfrac{5}{6}r=400-\tfrac{5}{6}(88)=327 \text{ lb. per lin. ft.}$$

Treating this value as a uniform load, the bending moment at the center is,

$$M_w=\frac{327l^2}{8}=51{,}900 \text{ ft. kips.}$$

Dividing by the truss width 34.5 ft., we obtain the chord stress $=\pm 1500$ kips at mid-span. The wind stresses at other sections will be proportional to parabolic ordinates, being zero at the ends of the span.

The shears in the lateral system may also be calculated for the resultant uniform load of 327 lb. per lin. ft.

EXAMPLE 2

Calculations for Two-hinged Suspension Bridge with Suspended Side Spans (Type 2S)

1. Dimensions.—The following dimensions are given:

l = Main Span = 1080 ft. $(l' = l)$.

l_1 = Side Span = 360 ft. $r = \frac{l_1}{l} = \frac{1}{3}$.

l_2 = Distance, tower to anchorage = 400 ft.

f = Cable-sag in main span = 108 ft. $n = \frac{f}{l} = \frac{1}{10}$.

f_1 = Cable-sag in side spans = 12 ft. $n_1 = \frac{f_1}{l_1} = \frac{1}{30}$. $v = \frac{f_1}{f} = \frac{1}{9}$.

d = Depth of Stiffening Truss = 22.5 ft.

Mean Chord Section (gross):
 Main Span, Top = 83, Bottom = 137 sq. in.
 Side Spans, Top = 52, Bottom = 52 sq. in.

I, (Main Span) $= 83(14)^2 + 137(8.5)^2 = 26{,}200$ in.^{2} ft.^{2}

I_1, (Side Spans) $= 2 \times 52(11.25)^2 = 13{,}100$ in.^{2} ft.^{2}

$$i = \frac{I}{I_1} = 2.0.$$

Width, center to center of trusses or cables = 42.5 ft.

A = Cable Section = 78 sq. in. per cable. $(A_1 = A)$.

$\tan \alpha$ = Slope of Cable Chord in Main Span = 0.

$\tan \alpha_1$ = Slope of Cable Chord in Side Span $= 4(n - n_1) = .267$.

$\sec \alpha_1 = 1.034$.

2. Stresses in Cable.—(All values given per cable).
Given:

w = Dead Load (including cable) = 2385 lb. per lin. ft.

p' = Live Load = 860 lb. per lin. ft.

t = Temperature Variation = $\pm 60°$ F. $(E\omega t = 11{,}720$ lb. per sq. in.).

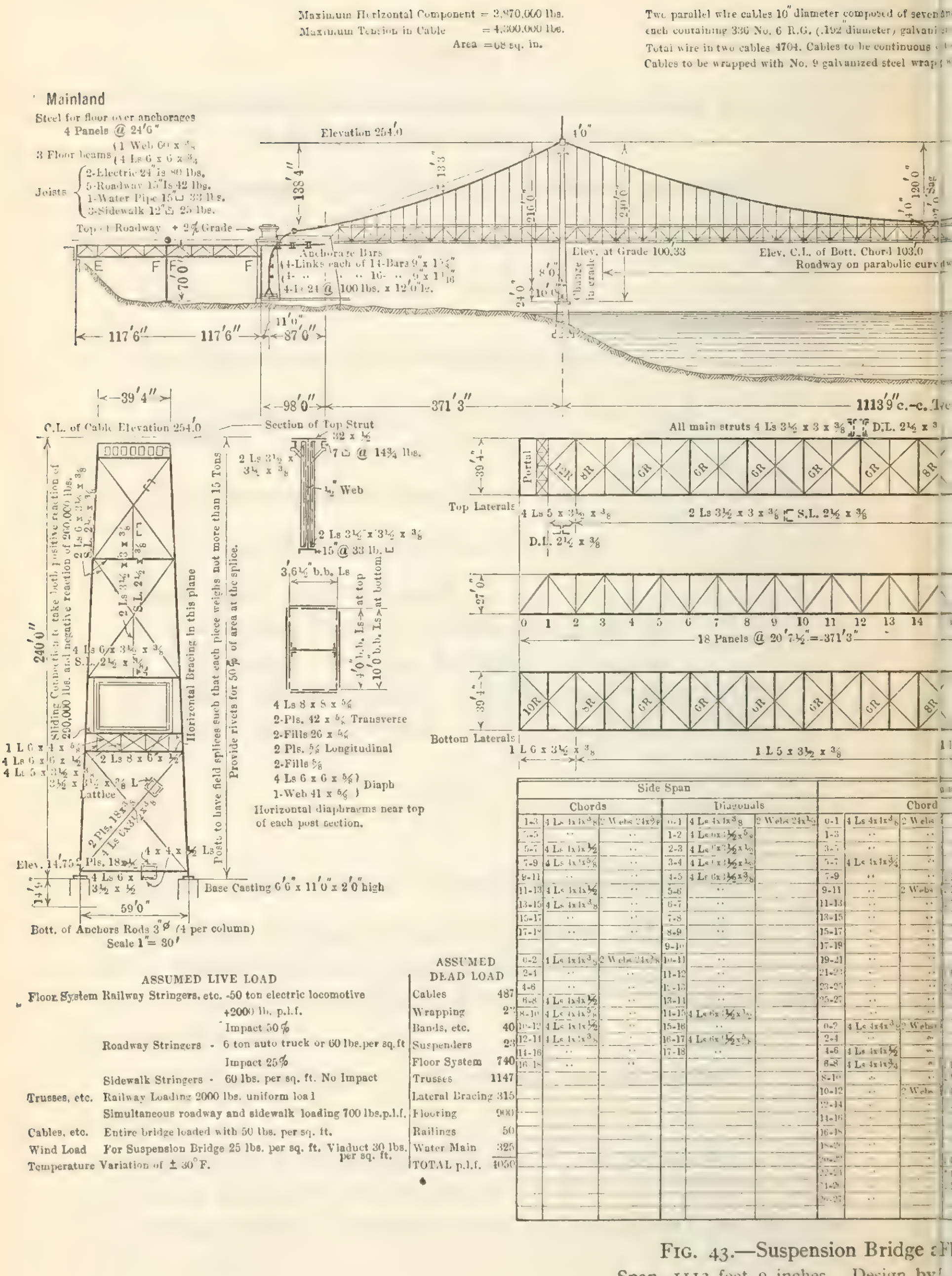
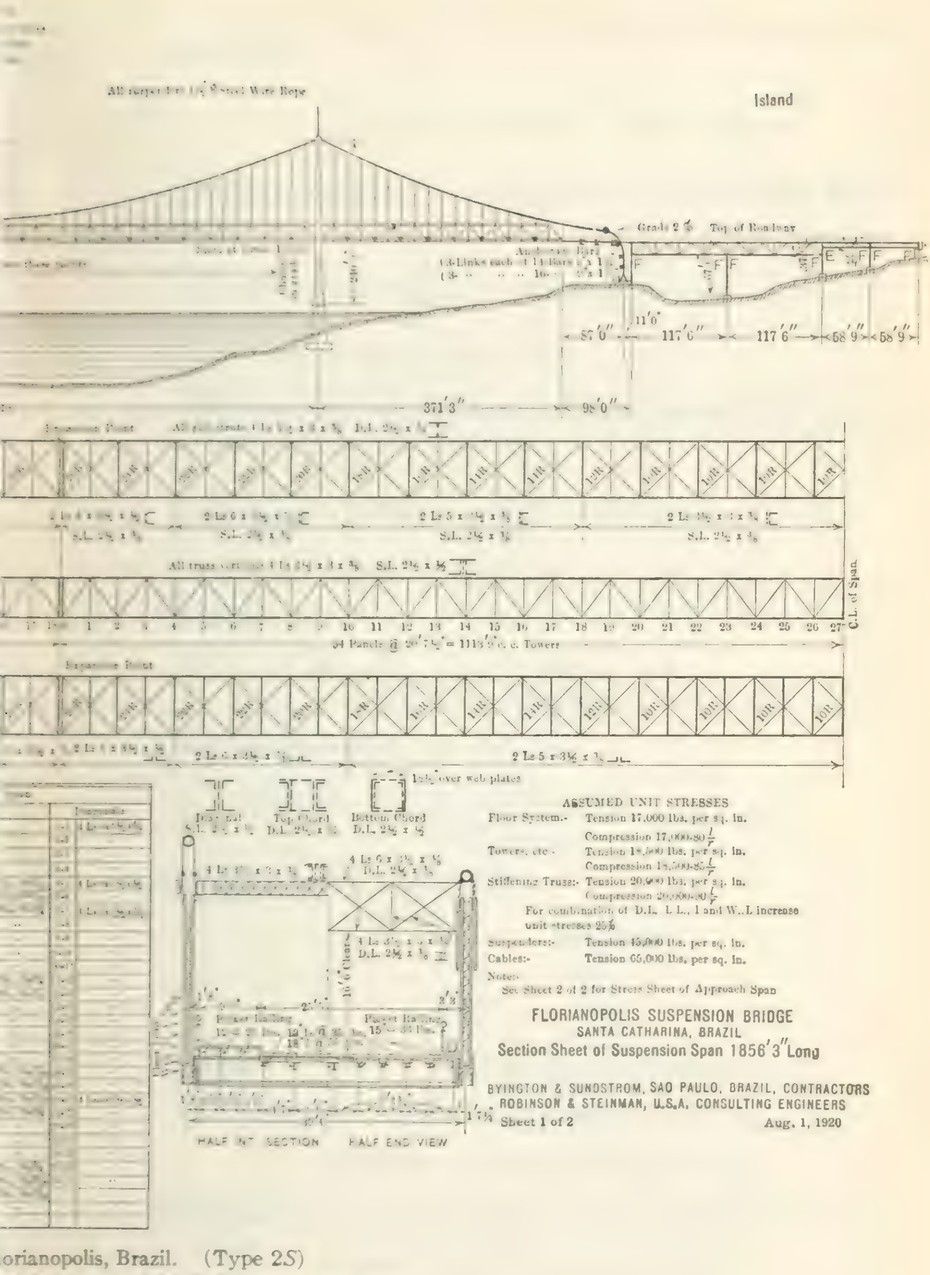


FIG. 43.—Suspension Bridge
Span, 1113 feet 9 inches. Design by



orianopolis, Brazil. (Type 2S)

D. Robinson and D. B. Steinman, 1920.

(To face page 134)

For Dead Load, by Eq. (11), the horizontal component of cable stress is,

$$H=\frac{wl^2}{8f}=\frac{10}{8}wl=3220 \text{ kips.} \quad (1 \text{ kip}=1000 \text{ lb.})$$

For Live Load, by Eq. (125), the denominator of the H-equation is,

$$N=\tfrac{8}{5}(1+2irv^2)+\frac{3I}{Af^2}\cdot\frac{l'}{l}(1+8n^2)+\frac{6I}{A_1f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1(1+8n_1{}^2)$$

$$=1.626+.093+.071=1.790.$$

By Eq. (135), the horizontal tension produced by live load, covering all three spans, will be,

$$H=\frac{1}{5Nn}(1+2ir^3v)p'l=\frac{2}{1.790}(1.0164)(9300)=1050 \text{ kips.}$$

The total length of cable between anchorages is given by Eq. (154):

$$\frac{L}{l}=\left(1+\frac{8}{3}n^2\right)+2\frac{l_2}{l}\left(\sec\alpha_1+\frac{8}{3}\,\frac{n_1{}^2}{\sec^3\alpha_1}\right)=1.027+.767=1.794.$$

Then, for temperature, by Eq. (156),

$$H_t=-\frac{3EI\omega tL}{f^2Nl}=\frac{3(11720)(26200)(1.794)}{(108)^2\times 1.790}=\mp 80 \text{ kips.}$$

Adding the values found for H:

D. L.	3220 kips
L. L.	1050
Temp.	80

we obtain, Total $H=4350$ kips per cable.

The maximum tension in the cable is, by Eq. (5),

$$T_1=H\cdot\sec\phi_1=H(1.08)=4700 \text{ kips.}$$

At 60,000 lb. per sq. in., the cable section required is:

$$4700\div 60=78 \text{ sq. in. per cable.}$$

3. Moments in Stiffening Truss—Main Span.—Given:

$$\text{Live Load} = p = 1600 \text{ lb. per lin. ft.}$$

(All values given and calculated are per truss.)

With the three spans completely loaded, the bending moment at any section x of the main span is given by Eq. (140):

$$\text{Total } M = \tfrac{1}{2}px(l-x)\left[1 - \frac{8}{5N}(1+2ir^3v)\right] = \tfrac{1}{2}px(l-x)[.091].$$

Hence only 9.1 per cent of the full live load is carried by the stiffening truss. Accordingly, at the center,

$$\text{Total } M = .091\frac{pl^2}{8} = +21.200 \text{ ft. kips.}$$

At other points, the values of M are proportional to the ordinates of a parabola. They are obtained as follows:

Section	Parabolic Coefficient	Total M
$\frac{x}{l}=0$	$4\times 0 \times 1 = 0$	0
0.1	$4\times .1 \times .9 = .36$	+7,600
0.2	$4\times .2 \times .8 = .64$	+13,600
0.3	$4\times .3 \times .7 = .84$	+17,800
0.4	$4\times .4 \times .6 = .96$	+20,400
0.45	$4\times .45 \times .55 = .99$	+21,000
0.5	$4\times .5 \times .5 = 1.00$	+21,200 ft. kips

For maximum and minimum moments, the critical points are found by solving Eq. (142):

$$C(k) = N \cdot n \cdot \frac{x}{y} = 0.179\frac{x}{y},$$

with the aid of Table I or Fig. 12 or 13:

$\frac{x}{l}$	$\frac{y}{l}$	$\frac{x}{y}$	$C(k)$	k	$D(k)$
0	0	(2.50)	(.448)	(.364)	(.508)
.1	.036	2.78	.498	.402	.411
.2	.064	3.12	.559	.448	.310
.3	.084	3.57	.640	.512	.202
.4	.096	4.17	.747	.603	.095
.45	.099	4.55	.815	.667	.050+.000
.5	.100	5.00	.895	.755	.016+.016
.55	.099	5.55	.995	.950	.000+.050

The values of $D(k)$, found from Table I or Fig. 13, are recorded in the above tabulation. For all sections from $x=\frac{N}{4}\cdot l=.447l$ to $x=.553l$, there are double values of $D(k)$, as explained under Eq. (143′).

The values of the minimum moments are then given by Eq. (143):

$$\text{Min. } M=-\frac{2px(l-x)}{5N}[D(k)+4ir^3v]$$

$$=-417{,}000\frac{x}{l}\left(1-\frac{x}{l}\right)[D(k)+.033],$$

and the maximum moments are then given by Eq. (144):

$$\text{Max. } M=\text{Total } M-\text{Min. } M.$$

Section	$\frac{x}{l}\left(1-\frac{x}{l}\right)$	[ ]	Minimum M	Total M	Maximum M
$\frac{x}{l}=0$	0	(.541)	0	0	0
.1	.09	.444	−16,700	+ 7,600	+24,300
.2	.16	.343	−22,900	+13,600	+36,500
.3	.21	.235	−20,600	+17,800	+38,400
.4	.24	.128	−12,800	+20,400	+33,200
.45	.248	.083	− 8,600	+21,000	+29,600
.5	.25	.065	− 6,800	+21,200	+28,000 ft. kips

Dividing the maximum and minimum moments by the truss depth ($d = 22.5$), we obtain the respective chord stresses. Adding the temperature and wind stresses found as in Example 1 and dividing by the specified unit stresses, we obtain the required chord sections.

4. Bending Moments in Side Spans.—

($p = 1600$ lb. per lin. ft. per truss).

With all three spans completely loaded, the bending moment at any section x_1 of either side span is given by Eq. (141):

$$\text{Total } M_1 = \tfrac{1}{2}px_1(l_1 - x_1)\left[1 - \frac{8}{5N}(1 + 2ir^3v)\frac{v}{r^2}\right] = \tfrac{1}{2}px_1(l_1 - x_1)[.091].$$

Accordingly, at the center,

$$\text{Total } M_1 = .091\,\frac{pl_1^2}{8} = +2300 \text{ ft. kips.}$$

There are no critical points for moments in the side spans. The minimum moments are given by Eq. (145):

$$\text{Min. } M_1 = -y_1 \cdot \frac{1 + ir^3v}{5Nn} \cdot pl = -y_1(1.124)pl.$$

Accordingly, at the center,

$$\text{Min. } M_1 = -12(1.124)(1730) = -23{,}400 \text{ ft. kips.}$$

The maximum moments are given by Eq. (146):

$$\text{Max. } M_1 = \text{Total } M_1 - \text{Min. } M_1.$$

Accordingly, at the center,

$$\text{Max. } M_1 = +2300 + 23{,}400 = +25{,}700 \text{ ft. kips.}$$

At other sections, the moments are proportional to the ordinates of a parabola:

Section	Parabolic Coefficient	Total M_1	Minimum M_1	Maximum M_1
$\frac{x_1}{l_1}=0$	0	0	0	0
.1	.36	+ 800	− 8,400	+ 9,200
.2	.64	+1500	−15,000	+16,500
.3	.84	+1900	−19,600	+21,500
.4	.96	+2200	−22,400	+24,600
.5	1	+2300	−23,400	+25,700 ft. kips

5. Shears in Stiffening Truss—Main Span.—

($p=1600$ lb. per lin. ft.)

With the three spans completely loaded, the shear at any section x of the main span is given by Eq. (147):

$$\text{Total } V=\tfrac{1}{2}p(l-2x)\left[1-\frac{8}{5N}(1+2ir^3v)\right]=\tfrac{1}{2}p(l-2x)[.091].$$

The shears will be the same as would be produced by loading the span with 9.1 per cent of the actual load, or with .091 $pl=157$ kips. Total $V=157\left(\frac{1}{2}-\frac{x}{l}\right)$.

Section	$\frac{1}{2}-\frac{x}{l}$	Total V
$\frac{x}{l}=0$	0.5	+79 kips
.1	0.4	+63
.2	0.3	+47
.3	0.2	+31
.4	0.1	+16
.5	0	0

The maximum shears are given by Eq. (149):

$$\text{Max. } V=\tfrac{1}{2}pl\left(1-\frac{x}{l}\right)^2\left[1-\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G\left(\frac{x}{l}\right)\right],$$

where the values of $G\left(\frac{x}{l}\right)$ are taken from Table I or Fig. 12. The shears are obtained as follows: ($\frac{1}{2}pl=864$ kips).

Section	$\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)$	$G\left(\frac{x}{l}\right)$	$[\ ]$	$\left(1-\frac{x}{l}\right)^2$	Maximum V
$\frac{x}{l}=0$	2.23	.400	.107	1	+ 92+194
.1	1.79	.482	.136	.81	+ 95+ 96
.2	1.34	.565	.243	.64	+134+ 22
.3	0.89	.647	.424	.49	+179
.4	.45	.726	.673	.36	+210
.5	0	.800	1.000	.25	+216 kips

For all sections, $x<\frac{l}{2}\left(1-\frac{N}{4}\right)=.277l$, the loading for maximum shear extends from the given section x to a critical point kl defined by Eq. (150):

$$C(k)=\frac{N}{4}\cdot\frac{l}{l-2x}=\frac{.446}{1-\frac{2x}{l}}.$$

The values $C(k)$ are solved for k and $G(k)$ with the aid of Fig. 13:

Section	$1-\frac{2x}{l}$	$C(k)$	k
0	.1	.446	.362
.1	.8	.558	.448
.2	.6	.744	.600

For these sections, a correction is to be added to the values of Max. V found above. This additional shear is given by Eq. (151):

$$\text{Add. } V=\tfrac{1}{2}pl(1-k)^2\left[\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G(k)-1\right].$$

Section	k	$(1-k)^2$	$\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)$	$G(k)$	$[\ ]$	Add. V
$\frac{x}{l}=0$	.362	.407	2.23	.696	.552	+194 kips
.1	.448	.305	1.79	.762	.365	+ 96
.2	.600	.160	1.34	.860	.160	+ 22

The minimum shears are then given by Eq. (153):

$$\text{Min. } V=\text{Total } V-\text{Max. } V.$$

Section	Total V	Maximum V	Minimum V
$\frac{x}{l}=0$	+79	+286 kips	−207 kips
.1	+63	+191	−128
.2	+47	+156	−109
.3	+31	+179	−148
.4	+16	+210	−194
.5	0	+216	−216

6. Shears in Side Spans.—

$$(p=1600 \text{ lb. per lin. ft.},\ l_1=360 \text{ ft.})$$

With the three spans completely loaded, the shear at any section x_1 in the side spans will be, by Eq. (148),

$$\text{Total } V_1=\tfrac{1}{2}p(l_1-2x_1)\left[1-\frac{8}{5N}\,\frac{v}{r^2}\,(1+2ir^3v)\right]=pl_1\left(\frac{1}{2}-\frac{x_1}{l_1}\right)[.091].$$

Since $l_1=\frac{1}{3}l$, these shears will be $\frac{1}{3}$ of the corresponding values in the main span.

Section	Total V_1
$\frac{x_1}{l_1}=0$	+26 kips
.1	+21
.2	+16
.3	+10
.4	+ 5
.5	0

There are no critical points for shear in the side spans. The maximum shear at any section x_1 is given by Eq. (152):

$$\text{Max. } V_1=\tfrac{1}{2}pl_1\left(1-\frac{x_1}{l_1}\right)^2\left[1-\frac{8}{N}irv^2\left(\frac{1}{2}-\frac{x_1}{l_1}\right)\cdot G\left(\frac{x_1}{l_1}\right)\right].$$

Section	$\frac{8}{N}(irv^2)\left(\frac{1}{2}-\frac{x_1}{l_1}\right)$	$G\left(\frac{x_1}{l_1}\right)$	$[\ \ ]$	$\left(1-\frac{x_1}{l_1}\right)^2$	Maximum V_1
$\frac{x_1}{l_1}=0$	.0183	.400	.993	1	+286 kips
.1	.0147	.482	.993	.81	+232
.2	.0110	.565	.994	.64	+183
.3	.0073	.647	.995	.49	+140
.4	.0037	.726	.997	.36	+103
.5	0	.800	1.000	.25	+ 72

The minimum shears in the side spans are given by Eq. (153′):

$$\text{Min. } V_1=\text{Total } V_1-\text{Max. } V_1.$$

Section	Minimum V_1
$\frac{x_1}{l_1}=0$	−260 kips
.1	−211
.2	−167
.3	−130
.4	− 98
.5	− 72

7. Temperature Stresses.—

$$(H_t=\mp 80 \text{ kips}).$$

The stresses in the main span from temperature variation are figured exactly as in Example 1 (Type $2F$), using Eqs. (157) and (158):

$$M_t=-H_t\cdot y,$$

$$V_t=-H_t(\tan\phi-\tan\alpha).\qquad(\text{Here, }\tan\alpha=0).$$

The temperature moments in the side spans are given by the formula:

$$M_t = -H_t \cdot y_1,$$

and will therefore be $v(=\frac{1}{9})$ times the corresponding main-span values.

Section	Parabolic Coefficient	M_t
$\frac{x_1}{l_1}=0$	0	0
.1	.36	±350
.2	.64	±610
.3	.84	±810
.4	.96	±920
.5	1	±960 ft. kips

The temperature shears in the side spans are given by the formula:

$$V_t = -H_t(\tan\phi_1 - \tan\alpha_1). \quad (\text{Here, } \tan\alpha_1 = .267.)$$

They will be $\frac{n_1}{n}(=\frac{1}{3})$ times the corresponding main-span values.

Section	$\tan\phi_1 - \tan\alpha_1$	V_t
$\frac{x_1}{l_1}=0$	$4n_1 = .133$	±11 kips
.1	.106	± 8
.2	.080	± 6
.3	.053	± 4
.4	.027	± 2
.5	0	0

8. Wind Stresses.—The wind stresses in the bottom chords and lateral bracing are calculated exactly as in Example 1 (Type $2F$).

The assumed wind load ($=p=400$ lb. per lin. ft.) is reduced by the fractional amount, at span center,

$$\frac{r}{p}=\frac{.013\dfrac{wl^4}{vEI}}{1+.013\dfrac{wl^4}{vEI}}=\frac{.173}{1+.173}=0.147.$$

($w=4770$ lb. per lin. ft.; $v=130$ ft.; $I=124{,}000$ in.^{2} ft.^{2})

Since the equivalent uniform value of r is $\frac{5}{6}$ of the mid-span value, the resultant horizontal load on the span is,

$$p-\tfrac{5}{6}r=400-\tfrac{5}{6}(59)=351 \text{ lb. per lin. ft.}$$

Treating this value as a uniform load, the bending moment at the center is,

$$M_w=\frac{351l^2}{8}=\pm 51{,}000 \text{ ft. kips.}$$

Dividing by the truss width, 42.5 ft., we obtain the chord stress $=\pm 1200$ kips at mid-span. The wind stresses at other sections will be proportional to parabolic ordinates.

The end shears in the lateral system will be:

$$V_w=\frac{351l}{2}=\pm 190 \text{ kips.}$$

In the side spans, unless they exceed 1000 feet in length, the reduction in effective wind pressure may be neglected. (In this example, $\frac{r}{p}$ would amount to only 1 per cent.) Hence the moments and shears are calculated for the full specified wind load of 400 lb. per lin. ft., acting on simple spans 360 ft. in length.

EXAMPLE 3

Calculations for Towers of Two-hinged Suspension Bridge (Type 2*S*)

1. Dimensions.—The bridge is the same as in Example 2.

Each tower consists of two columns of box section, stiffened with internal diaphragms, and rigidly tied together with transverse bracing in a vertical plane. Each tower column is 225 feet

high and is made of a double box section, 42.5 inches wide. The other dimension (d), parallel to the stiffening truss, is 4 feet at the top, increasing to 9 feet at the base. The walls are $1\frac{1}{4}$ inches thick (made up of $\frac{5}{8}$-inch plates and corner angles) and the vertical transverse diaphragm is $\frac{5}{8}$-inch thick. Splices are provided at such intervals as to keep the individual sections within specified limitations of length or weight for shipment. Horizontal diaphragms are provided at splices and, in general, at 10-foot intervals.

The tower columns are battered so as to clear the trusses. They are 42.5 feet center to center at the top and 53.5 feet center to center at the base.

2. Movement of Top of Tower.—The towers are assumed fixed at the base and the cable saddles immovable with respect to the tower.

The maximum fiber stress in the tower columns will occur when the live load covers the main span and the farther side span at maximum temperature. Under this condition of loading, the top of the tower will be deflected toward the main span, as a result of the following deformations:

1. The upward deflection (Δf_1) at the center of the unloaded side span.

2. The elongation of the cable between the anchorage and the tower, due to the elastic strain produced by the applied loads.

3. The elongation of the cable due to thermal expansion.

These deformations are computed as follows:

(Live Load $= p' = 860$ lb. per lin. ft. $H = 1040$ kips.)

1. The upward deflection Δf_1 is found by considering the unloaded side span as a simple beam subjected to an upward loading equal to the live-load suspender tensions (Eq. 78):

$$s = \frac{8f}{l^2} \cdot H = \frac{8}{10} \cdot \frac{1040}{1080} = 770 \text{ lb. per lin. ft. per truss,}$$

$$\Delta f_1 = \frac{5}{384} \frac{s l_1^4}{E I_1} = 0.428 \text{ ft.}$$

2. The elastic elongation of the cable in the side span is, by Eq. (55),

$$\Delta L_1 = \frac{Hl_2}{EA}(1 + \tfrac{16}{3}n_1^2 + \tan^2 \alpha_1) = .178(1.077) = .192 \text{ ft.}$$

3. The temperature expansion of the cable in the side span is, by Eqs. (53) and (26),

$$\Delta L_1 = \omega t l_2 \left(\sec \alpha_1 + \frac{8}{3} \frac{n_1^2}{\sec^3 \alpha_1} \right) = .156(1.037) = 0.162 \text{ ft.}$$

We also have:

$$\frac{\Delta L_1}{\Delta l_1} = \sec \alpha_1 + \frac{8}{3} \frac{n_1^2}{\sec^3 \alpha_1} = 1.037,$$

$$\frac{\Delta L_1}{\Delta f_1} = \frac{16}{3} \frac{n_1}{\sec^3 \alpha_1} = 0.160.$$

The deflection of the top of the tower is then given by,

$$y_0 = \Delta l_1 = \frac{\Delta l_1}{\Delta L_1} \cdot \frac{\Delta L_1}{\Delta f_1} \cdot \Delta f_1 + \frac{\Delta l_1}{\Delta L_1} \cdot \Sigma(\Delta L_1).$$

Substituting the values just calculated, we obtain the maximum tower deflection:

$$y_0 = \frac{.160}{1.037}(.428) + \frac{1}{1.037}(.192 + .162) = 0.408 \text{ ft.}$$

3. Forces Acting on Tower.—Considering this deflection as produced by an unbalanced horizontal force P applied at the top of the tower, this force may be calculated, if the sectional dimensions of the tower are known, by the formula,

$$y_0 = \frac{P}{E} \cdot \Sigma \left(\frac{x^2}{I} \cdot \Delta x \right).$$

In the present case, we find $\Sigma \dfrac{x^2}{I} \cdot \Delta x = 1740$. Hence,

$$P = y_0 \cdot \frac{E}{1740} = 17{,}200 y_0 = 7000 \text{ lb. per column.}$$

The other loads acting on the tower are the vertical reaction (V) at the saddles, and the end-shears (V_1) at the point of support of the stiffening truss. The saddle reaction is given by the formula:

$$V = 2H \cdot \tan \phi = 2 \times 4340 \times 0.4 = +3470 \text{ kips per column.}$$

The truss reaction, with all spans loaded and maximum temperature rise, is,

$$V_1 = (42+32)+(14+11) = +99 \text{ kips per column.}$$

With one side span unloaded, as assumed above,

$$V_1 = (45+32)+(11-140) = -52 \text{ kips per column.}$$

The inaccuracy introduced by neglecting this uplift, V_1, will be on the side of safety; therefore the column need be figured only for the horizontal load P and the vertical load V.

At any section x of the tower, the horizontal deflection (y) from the initial vertical position of the axis is given with sufficient accuracy by the equation for the elastic curve of the cantilever:

$$y = y_0\left[1 - \frac{3}{2}\left(\frac{x}{h}\right) + \frac{1}{2}\left(\frac{x}{h}\right)^3\right].$$

4. Calculation of Stresses.—The resulting extreme fiber stresses at any section of the tower will be:

$$\text{Combined Stress} = \frac{V}{A} + \frac{Pxc}{I} + \frac{V(y_0 - y)c}{I}.$$

The computations may be arranged as follows, the stresses being figured for convenience at 25-foot intervals:

Joint No.	x ft.	$y_0 - y$ ft.	$d = 2c$ ft.	A sq. in.	I in.^{2} ft.^{2}	$\frac{x^2}{I}$	$\frac{V}{A}$	$\frac{Pxc}{I}$	$\frac{V(y_0-y)c}{I}$	Combined Stresses lb./sq. in.
0	0	0	4.0	280	560	0	12,400	0	0	12,400
1	25	.068	4.5	295	730	.86	11,800	500	700	13,000
2	50	.134	5.0	310	940	2.66	11,200	900	1100	13,200
3	75	.197	5.5	325	1170	6.40	10,700	1200	1600	13,500
4	100	.254	6.0	340	1440	6.94	10,200	1500	1800	13,500
5	125	.305	6.5	355	1740	8.98	9,800	1600	2000	13,400
6	150	.348	7.0	370	2080	10.80	9,400	1800	2000	13,200
7	175	.380	7.5	385	2460	12.42	9,000	1900	2000	12,900
8	200	.400	8.0	400	2880	13.88	8,700	1900	1900	12,500
9	225	.408	9.0	430	3850	13.20	8,100	1800	1700	11,600
						$\Sigma = 69.54$				

5. Wind Stresses.—To the above tower stresses produced by live load and temperature, must be added the stresses due to wind loads.

The truss wind load of 400 lb. per lin. ft. produces a horizontal reaction at each tower of,

$$360\frac{l}{2}+400\frac{l_1}{2}=266 \text{ kips.}$$

This acts at Joint No. 4, ($x=100$).

The deflection of the stiffening truss under wind load produces a horizontal reaction at the top of each tower of $40\frac{l}{2}$; and the wind on the surface of the cables produces an addition to this reaction amounting to $10\left(\frac{l}{4}+\frac{l_1}{2}\right)$; hence the total reaction at the tower top $=26$ kips.

The wind acting directly on the tower is assumed at 25 lb. per sq. ft. of vertical elevation. This produces, at each joint, an equivalent concentrated load of $25\times(25d)$.

Joint No.	x ft.	d ft.	Wind Load kips	Shear kips	Moment ft. kips	Column Dist. ft.	A sq. in.	Stress from W. L. lb./sq. in.	Stress from L. L.+ Temp. lb./sq. in.	Total Stress lb./sq. in.
0	0	4	26+1	0	0	42.5	280	0	12,400	12,400
1	25	4.5	3	− 27	− 675	43.5	295	100	13,000	13,100
2	50	5	3	− 30	− 1,425	44.5	310	100	13,200	13,300
3	75	5.5	3	− 33	− 2,250	46.5	325	100	13,500	13,600
4	100	6	266+4	− 36	− 3,150	48.5	340	200	13,500	13,700
5	125	6.5	4	−306	−10,800	49.5	355	600	13,400	14,000
6	150	7	4	−310	−18,550	50.5	370	1000	13,200	14,200
7	175	7.5	5	−314	−26,400	51.5	385	1300	12,900	14,200
8	200	8	5	−319	−34,375	52.5	400	1600	12,500	14,100
9	225	0	3	−324	−42,475	53.5	430	1800	11,600	13,400

The bending moments, divided by the column distance, give the column stresses, and these divided by the areas give the unit stresses from wind load.

The transverse bracing is proportioned to resist the shears tabulated above.

EXAMPLE 4

Estimates of Cable and Wrapping

1. Calculation of Cable Wire.—Given a suspension bridge in which a cable section of 68 sq. in. is to be provided. To find the material required for cables and wrapping. Other data as in Example 2.

The total length of each cable is given by Eq. (154):

$$L = l(1+\tfrac{8}{3}n^2) + 2l_1\left(\sec\alpha_1 + \frac{8}{3}\,\frac{n_1^2}{\sec^3\alpha_1}\right)$$

$$= 1080(1.027) + 720(1.034 + .003) = 1110 + 746 = 1856 \text{ ft.}$$

To this must be added 43 ft. of cable at each end, between end of truss span and anchorage eyebars (scaled from drawing); hence,

$$\text{Total } L = 1856 + 86 = 1942 \text{ ft. per cable.}$$

No. 6 galvanized cable wire will be used = 0.192 in. diameter = .029 sq. in. area. Each cable consists of 7 strands of 336 wires each = 2352 wires at 0.29 sq. in. = 68 sq. in.

Weight of No. 6 galvanized wire = 0.1 lb. per ft.

Total cable wire = 2 × 2352 wires at 1942 ft. = 9,150,000 lin. ft.

Total weight of cable wire = 9,150,000 ft. at 0.1 lb. = 915,000 lb.

2. Calculation of Cable Diameter.—The diameter of the cable is figured as follows: The area of a strand will be 10 per cent greater than the aggregate section of the wires composing it. In this case the area of each strand will be,

$$110 \text{ per cent} \times \tfrac{68}{7} = 10.7 \text{ sq. in.}$$

The corresponding diameter is 3.7 in. The cable diameter will be 3 strand diameters = 11.1 inches. (Adding the thickness of wrapping, the finished cable will be 11.4 inches in diameter.)

3. Calculation of Wrapping Wire.—The wrapping consists of No. 9 galvanized wrapping wire (soft, annealed), weighing .06 lb. per ft. Deducting lengths of cable bands, etc., there will be 3250 ft. of cable to be wrapped. Since the wrapping wire is 0.15 in. diameter, it will make 80 turns per lin. ft. The diameter

of the cable is 11.1 inches, hence the length of each turn will be 2.9 ft.

Length of wrapping wire = 80 turns at 2.9 ft.
= 232 ft. per lin. ft. of cable.

Weight of wrapping wire = 232 ft. at .06 lb.
= 13.44 lb. per lin. ft. of cable.

Total wrapping wire = 3250 ft. of cable at 13.44 lb.
= 44,000 lb.

4. Estimate of Rope Strand Cables.—Instead of building the cable of individual wires, manufactured rope strands may be used. In the case at hand, with a factor of safety of 3, there would be required sixty-one $1\frac{5}{8}$-inch strands per cable. These galvanized steel ropes weigh 4.34 lb. per ft.; hence the total weight in the cables would be,

$$2 \times 1942 \text{ ft.} \times 61 \text{ strands at } 4.34 \text{ lb} = 1{,}030{,}000 \text{ lb.}$$

The diameter of the resulting cable would be $7 \times 1\frac{5}{8}$-in. = 11.4 in., plus the wrapping. (If rope strands are used, it should be remembered that their modulus of elasticity E is only about 20,000,000, as compared with 30,000,000 for parallel wire cables.)

EXAMPLE 5

Analysis of Suspension Bridge with Continuous Stiffening Truss (Type 0S)

(See Chap. I.. Pages 53 to 63.)

1. Dimensions.—The following dimensions are given:

$$l = \text{Main Span} = 40 \text{ panels at } 17'\ 7\tfrac{1}{2}'' = 705 \text{ ft.} \quad (l' = l).$$

$$l_2 = l_1 = \text{Side Span} = 10 \text{ panels at } 17'\ 7\tfrac{1}{2}'' = 176.25 \text{ ft.} \quad \left(r = \frac{l_1}{l} = \frac{1}{4}\right).$$

$$f = \text{Cable sag in Main Span} = 74.285 \text{ ft.} \quad \left(n = \frac{f}{l} = \frac{1}{9.5}\right).$$

f_1 = Cable-sag in Side Spans = 4.65 ft.

$$\left(n_1=\frac{f_1}{l_1}=\frac{1}{38}\right). \quad \left(v=\frac{f_1}{f}=\frac{1}{16}\right).$$

d = Depth of Truss = 15.′0 at towers, 10.′833 at center, 11.′346 at ends.

Width, center to center of trusses or cables = 27 ft.

I = Mean Moment of Inertia in main span = 1642 in.^{2} ft.^{2} (per truss).

I_1 = Mean Moment of Inertia in side spans = 2278.

$$\left(i=\frac{I}{I_1}=0.72\right).$$

A = Cable Section in main span = 7 strands of 282 wires at 0.192 in. diameter = 57.2 sq. in. per cable.

A_1 = Cable Section in side spans = A, + 2 strands of 76 wires = 61.6 sq. in. per cable.

tan α = Slope of Cable Chord in main span = .026.

tan α_1 = Slope of Cable Chord in side spans = 0.5.

$$e=\text{Coefficient of Continuity}=\frac{2+2irv}{3+2irv}=0.602$$

2. Stresses in Cables.—(All values per cable).

For dead load (w = 2850 lb. per lin. ft. per cable), the horizontal tension is given by Eq. (11):

$$H=\frac{wl^2}{8f}=\frac{9.5}{8}wl=2380 \text{ kips per cable.}$$

The denominator for other values of H is given by Eq. (203):

$$N=\tfrac{8}{5}-4e+3e^2+2ir(\tfrac{8}{5}v^2+e^2-2ev)$$
$$+\frac{3I}{Af^2}\cdot\frac{l'}{l}(1+8n^2)+\frac{6I}{A_1f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1(1+8n_1^2)=0.413.$$

With the live load (p = 750 lb. per lin. ft. per cable) covering the main span, Eq. (206) gives,

$$H=\frac{1}{Nn}\left(\frac{1}{5}-\frac{e}{4}\right)\cdot pl=\frac{.475}{N}pl=610 \text{ kips per cable.}$$

With the live load ($p_1=750$ lb. per lin. ft. per cable) covering both side spans, Eq. (207) gives,

$$H=\frac{2ir^3}{N\cdot n}\left(\frac{v}{5}-\frac{e}{8}\right)\cdot p_1 l=-\frac{.0134}{N}\cdot p_1 l=-17 \text{ kips.}$$

The total length of cable is given by Eq. (154):

$$\frac{L}{l}=(1+\tfrac{8}{3}n^2)+2\frac{l_1}{l}\left(\sec\alpha_1+\frac{8}{3}\frac{n_1^2}{\sec^3\alpha_1}\right)=1.8542.$$

Substituting this value in Eq. (214), we obtain the cable tension produced by temperature variation ($t=\pm 60°$ F., $E\omega t=11{,}720$):

$$H_t=-\frac{3EI\omega tL}{f^2\cdot N\cdot l}=\mp 47 \text{ kips per cable.}$$

Combining the values for dead load, main-span live load, and fall in temperature, we obtain,

$$\text{Max. } H=3037 \text{ kips per cable.}$$

In the main span, the maximum slope of the cable is $\tan\phi=\tan\alpha+4n=.447$; $\sec\phi=1.096$. For this slope, the stress in the cable is, by Eq. (5),

$$\text{Max. } T=H\cdot\sec\phi=3330 \text{ kips.}$$

At 60,000 lb. per sq. in., the cable section required is $3330\div 60=55.5$ sq. in. (The section provided is $A=57.2$ sq. in.)

In the side spans, the maximum slope of the cable is $\tan\phi_1=\tan\alpha_1+4n_1=.605$; $\sec\phi_1=1.17$. For this slope, the stress in the cable is, by Eq. (5),

$$\text{Max. } T_1=H\cdot\sec\phi_1=3550 \text{ kips.}$$

At 60,000 lb. per sq. in., the cable section required is $3550\div 60=59.2$ sq. in. (The section provided is $A_1=61.6$ sq. in.)

3. Influence Line for H.—For a concentration P traversing the main span, the values of H are given by Eq. (204):

$$H=\frac{1}{N\cdot n}[B(k)-\tfrac{3}{2}e(k-k^2)]\cdot P.$$

Taking the values of $B(k)$ from Table I or Fig. 12, we obtain the following main-span influence ordinates for H:

Load Position	Ordinate $\frac{H}{P}$
$k=0$	0
0.1	.387
0.2	.948
0.3	1.482
0.4	1.860
0.5	2.000

For a concentration P_1 traversing either side span, the values of H are given by Eq. (205):

$$H=\frac{ir^2}{N\cdot n}\left[v\cdot B(k_1)-\frac{c}{2}(k_1-k_1^3)\right]\cdot P_1.$$

where k_1l_1 is measured from the free end of the span. Substituting the values of $B(k_1)$, we obtain the following side-span influence ordinates for H:

Load Position	Ordinate $\frac{H}{P_1}$
$k_1=0$	0
0.2	−.048
0.4	−.085
0.6	−.100
0.8	−.078
1.0	0

4. Bending Moments in Main Span.—The bending moments will be obtained by the method of unit loads applied at successive panel points, using Eq. (182):

$$M=\left(M_0+\frac{l-x}{l}\cdot M_1+\frac{x}{l}\cdot M_2\right)-H(y-ef)=M'-H(y-ef).$$

In this case, $ef=44.7$ ft., and the values of $(y-ef)$ are as follows:

Panel Point	$\frac{x}{l}$	$y-ef$
No. 20	0	-44.7 ft.
16	.1	-17.9
12	.2	$+\ 2.8$
8	.3	$+17.7$
4	.4	$+26.4$
0	.5	$+29.6$

NOTE.—In this bridge, the panel points were numbered consecutively in both directions, starting with No. 0 at the middle of the span; No. 20 is at the towers, and No. 30 at the free ends.

The value of H is a constant for each load position and is taken from the influence table figured above.

For each load position, the moments M_1 and M_2 at the towers are given by Eqs. (190) and (191):

$$M_1 = -Pl\cdot k(1-k)\frac{(3+2ir)(1-k)+2ir}{(3+2ir)(1+2ir)}$$

$$= -[55.5+516(1-k)]P\cdot k(1-k).$$

$$M_2 = -Plk(1-k)\frac{(3+2ir)k+2ir}{(3+2ir)(1+2ir)}$$

$$= -[55.5+516k]P\cdot k(1-k).$$

The bending moment M' at the section carrying the load is,

$$M' = M_0 + M_1(1-k) + M_2k = Pk(1-k)l + M_1(1-k) + M_2k.$$

Using the three above equations, we obtain the following controlling values of M' for a unit load $P=1$.

Load Position	M_1 at Near Tower	M' at Load	M_2 at Far Tower
$k=0$	0	0	0
.1	-46.6	$+20.5$	$-\ 9.7$
.2	-75.0	$+47.7$	-25.4
.3	-87.7	$+73.9$	-44.2
.4	-87.7	$+91.3$	-62.8
.5	-78.4	$+97.8$	-78.4

These values give the three vertices of the equilibrium triangle; and, for each load position, the values of M' for other sections may be tabulated by straight-line interpolation. Subtracting from each value of M' the corresponding value of $H(y-cf)$, we obtain the unit-load bending moments M. A typical tabulation for this computation is as follows:

Unit Load at Panel Point 12. ($k=0.2$). ($H=.948$)

Panel Point	M'	$H(y-cf)$	M
No. 20	-75.0*	-42.4	-32.6
16	-13.7	-17.0	$+3.3$
12	$+47.7$*	$+2.7$	$+45.0$
8	$+38.6$	$+16.8$	$+21.8$
4	$+29.4$	$+25.1$	$+4.3$
0	$+20.3$	$+28.1$	-7.8
4	$+11.2$	$+25.1$	-13.9
8	$+2.0$	$+16.8$	-14.8
12	-7.2	$+2.7$	-9.9
16	-16.3	-17.0	$+0.7$
20	-25.4*	-42.4	$+17.0$

With the left side-span completely loaded (unit load at each panel point), Eqs. (197), (198) and (207) give:

$$M_1=-\frac{p_1l^2}{4}\cdot\frac{2ir^3(1+ir)}{(3+2ir)(1+2ir)}=-.001453p_1l^2=-41.0$$

$$M_2=+\frac{p_1l^2}{4}\cdot\frac{ir^3}{(3+2ir)(1+2ir)}=+.000616p_1l^2=+17.4.$$

$$H=\frac{ir^3}{Nn}\left(\frac{v}{5}-\frac{e}{8}\right)\cdot p_1l=-.0017\frac{pl^2}{f}\qquad=-0.645.$$

The resulting bending moments in the main span will be, by Eq. (182),

$$M=\frac{l-x}{l}M_1+\frac{x}{l}\cdot M_2-H(y-cf),$$

and are obtained by a tabulation similar to the one above.

The influence values of M obtained in the series of tabulations just described may be summarized as follows (only every fourth panel point shown here):

INFLUENCE VALUES OF M FOR UNIT LOADS

Load Position	M at Panel Point					
	20	16	12	8	4	0
Panel point No. 20	0	0	0	0	0	0
16	−29.3	+27.4	+16.0	+6.8	+0.1	−4.6
12	−32.6	+3.3	+45.0	+21.8	+4.3	−7.8
8	−21.3	−7.4	+15.4	+47.0	+17.1	−4.1
4	−4.6	−9.6	−3.3	+13.8	+41.8	+10.6
0	+11.0	−7.4	−13.5	−8.1	+9.4	+38.6
4	+20.3	−3.9	−16.8	−18.8	−9.5	+10.6
8	+22.2	−0.8	−14.8	−20.1	−16.4	−4.1
12	+18.7	+0.7	−0.0	−14.8	−13.9	−7.8
16	+7.6	+0.4	−4.3	−6.8	−6.7	−4.6
20	0	0	0	0	0	0
Left side span	+12.3	+23.7	+31.3	+35.0	+34.9	+31.0
Right side span	−46.3	−23.1	−3.9	+11.6	+23.1	+31.0
Maximum M	+323.1	+135.4	+317.4	+379.8	+323.6	+275.8
Minimum M	−411.4	−139.8	−245.1	−270.6	−184.3	−127.4

Max. M is the summation of all positive influence values, and Min. M is the summation of all negative influence values.

These results are multiplied by the panel load $\left(P=\frac{pl}{40}=13.22\right.$ kips) to obtain the bending moments in foot-kips; and the latter values are divided by the truss depths at the respective panel points to obtain the chord stresses in kips.

The temperature moments are given by Eq. (215):

$$M_t = -H_t(y-ef),$$

where $H_t = \mp 47$ kips; the values of $(y-ef)$ have been tabulated above. These moments are combined with the live-load moments as the specifications may prescribe.

5. Shears in Main Span.—The shears in the main span are given by Eq. (188):

$$V = V_0 + \frac{M_2 - M_1}{l} - H(\tan\phi - \tan\alpha) = V' - H(\tan\phi - \tan\alpha).$$

The method of unit loads will be used. The values of H, M_2 and M_1 have been calculated above for different load positions.

Load Position	V_0	$\frac{M_2 - M_1}{l}$	V' (to left of load)	V'' (to right of load)	H
Panel point No. 20...	1.0	0	+1.0	0	0
16...	.9	.020	+ .920	−.080	.387
12...	.8	.067	+ .867	− .133	.948
8...	.7	.092	+ .792	−.208	1.482
4	.6	.066	+ .666	−.334	1.860
0...	.5	0	+ .500	−.500	2.000

The value of $(\tan\phi - \tan\alpha)$ decreases uniformly from $4n = .42$ at the left tower (Panel Pt. No. 20) to 0 at the center (Panel Pt. No. 0) and to $-4n = -.42$ at the right tower (Panel Pt. No. 20).

Substituting these values in the above formula for V, an influence table for shears is constructed, similar to the preceding influence table for moments. (Only every fourth panel point shown here):

Influence Values of M for Unit Loads

Load Position	V at Panel Point					
	20	16	12	8	4	0
Panel point No. 20...	+1.00	0	0	0	0	0
16.....	+ .76	− .21	− .18	− .15	− .11	− .08
12.....	+ .47	+ .55	− .37	− .29	− .21	− .13
8.....	+ .17	+ .29	+ .42	− .46	− .33	− .21
4.....	− .11	+ .05	+ .20	+ .36	− .49	− .33
0.....	− .34	− .17	0	+ .16	+ .34	± .50
4.....	− .45	− .29	− .14	+ .02	+ .17	+ .33
8.....	− .41	− .29	− .16	− .04	+ .09	+ .21
12.....	− .27	− .19	− .11	− .03	+ .05	+ .13
16.....	− .08	− .05	− .02	+ .01	+ .05	+ .08
20.....	0	0	0	0	0	0
Left side span..........	+ .35	+ .30	+ .24	+ .19	+ .13	+ .08
Right side span........	+ .19	+ .14	+ .08	+ .03	− .03	− .08
Maximum V..........	+8.02	+4.74	+3.42	+2.94	+3.43	+4.56
Minimum V.	−6.58	−4.58	−3.60	−3.38	−4.12	−4.56

Max. V is the summation of all positive influence values, and Min. V is the summation of all negative influence values. These results are multiplied by the panel load $\left(P=\frac{pl}{40}=13.22 \text{ kips}\right)$ to obtain the vertical shears in kips.

The temperature shears are given by Eq. (217):

$$V_t=-H_t(\tan\phi-\tan\alpha).$$

The shears are then multiplied by the respective secants of inclination, to obtain the stresses in the web members of the stiffening truss.

6. Bending Moments in Side Spans.—The bending moments in the side spans are obtained by the method of unit loads, using Eq. (183):

$$M=M_0+\frac{x_1}{l_1}M_1-H\left(y_1-\frac{x_1}{l_1}\cdot ef\right)=M'-Hy'.$$

For loads in the main span, $M_0=0$, and the values of M_1 and M_2 are the same as calculated above. For the far side span completely loaded, $M_0=0$, and the value of M_1 is the same as the value of M_2 calculated above. For unit loads $(P=1)$ in the given side span, the moment M_1 is given by Eq. (194):

$$M_1=-Pl\frac{2ir^2(1+ir)(k_1-k_1^3)}{(3+2ir)(1+2ir)}=16.4P(k_1-k_1^3),$$

and

$$M_0=Pl_1(k_1-k_1^2).$$

The values of H will be the same as calculated above. Accordingly, we have the following values for a unit load $(P=1)$ traversing the side span.

Load Position	k_1	M'	H
Panel point No. 20......	1.0	0	0
22......	.8	+24.4	−.048
24......	.6	+38.5	−.085
26......	.4	+40.1	−.100
28......	.2	+27.6	−.078
30......	0	0	0

The values of $y' = y_1 - \frac{x_1}{l_1} \cdot ef$ are as follows:

Panel Point:	20	22	24	26	28	30
$\frac{x_1}{l_1} =$	1.0	.8	.6	.4	.2	0
$y' =$	-44.7	-35.8	-26.8	-17.9	-8.9	0

Substituting the various values in the equation:

$$M = M' - H \cdot y',$$

we obtain the following influence table for side-span moments (only every second panel point shown here):

INFLUENCE VALUES OF M FOR UNIT LOADS

Load Position	M at Panel Point						
	20	21	22	24	26	28	30
Panel point No. 20.	0	0	0	0	0	0	0
22.	− 6.8	+ 8.0	+ 22.8	+ 17.2	+ 11.6	+ 5.8	0
24.	− 10.1	+ 1.6	+ 13.3	+ 36.6	+ 24.6	+ 12.3	0
26.	− 10.0	− 1.7	+ 6.4	+ 22.7	+ 38.8	+ 19.5	0
28.	− 6.7	− 2.3	+ 1.9	+ 10.5	+ 18.9	+ 27.1	0
30.	0	0	0	0	0	0	0
Maximum M	0	+ 27	+ 88	+179	+193	+130	0
Minimum M	− 70	− 8	0	0	0	0	0
Far side span.	− 12	− 9	− 7	− 4	− 2	− 1	0
Main span.	+311	+226	+167	+ 71	+ 14	+ 4	0
	−365	−356	−349	−318	−247	−153	0
Total Maximum M.	+311	+253	+255	+250	+207	+134	0
Total Minimum M.	−447	−373	−356	−322	−249	−154	0

The above results are to be multiplied by the panel load $\left(P = \frac{pl}{40} = \frac{pl_1}{10} = 13.22 \text{ kips}\right)$ to obtain the maximum and minimum bending moments in the side span.

The temperature moments are calculated by Eq. (216):

$$M_t = -H_t\left(y_1 - \frac{x_1}{l_1}\cdot cf\right) = -H_t\cdot y',$$

where $H_t = \mp 47$ kips.

7. Shears in Side Spans. The left side-span shears are calculated by Eq. (189):

$$V = \left(V_0 + \frac{M_1}{l_1}\right) - H\left(\tan\phi_1 - \tan\alpha_1 - \frac{cf}{l_1}\right) = V' - K\cdot H.$$

At the tower (Panel Point 20), $K = -4n_1 - \frac{cf}{l_1} = -.105 - .254 = -.359$; and the value of K diminishes uniformly to $K = +4n_1 - \frac{cf}{l_1} = +.105 - .254 = -.149$ at the free end (Panel Point 30).

Substituting in the above formula the known values of H, M_1 and K, we obtain the following table of influence ordinates for side-span shears (only every second panel point shown here):

INFLUENCE VALUES OF V FOR UNIT LOADS

Load Position	V at Panel Point					
	20	22	24	26	28	30
Panel point No. 20	0	0	0	0	0	0
22	− .80	− .81	+ .19	+ .19	+ .18	+ .18
24	− .60	− .61	− .61	+ .38	+ .38	+ .38
26	− .40	− .41	− .41	− .41	+ .59	+ .58
28	− .20	− .21	− .21	− .21	− .21	+ .79
30	0	0	0	0	0	0
Maximum V	0	+0.1	+0.6	+1.4	+2.7	+4.4
Minimum V	−4.5	−3.7	−2.2	−1.1	−0.3	0
Far side span	+0.3	+0.3	+0.3	+0.2	+0.2	+0.2
Main span	+5.4	+3.8	+2.1	+1.2	+0.3	0
	−0.9	−1.1	−1.6	−2.5	−3.4	−5.1
Total Maximum V	+5.7	+4.2	+3.0	+2.8	+3.2	+4.6
Total Minimum V	−5.4	−4.8	−3.8	−3.6	−3.7	−5.1

These results are to be multiplied by the panel load ($P = 13.22$ kips) to obtain the maximum and minimum shears in kips.

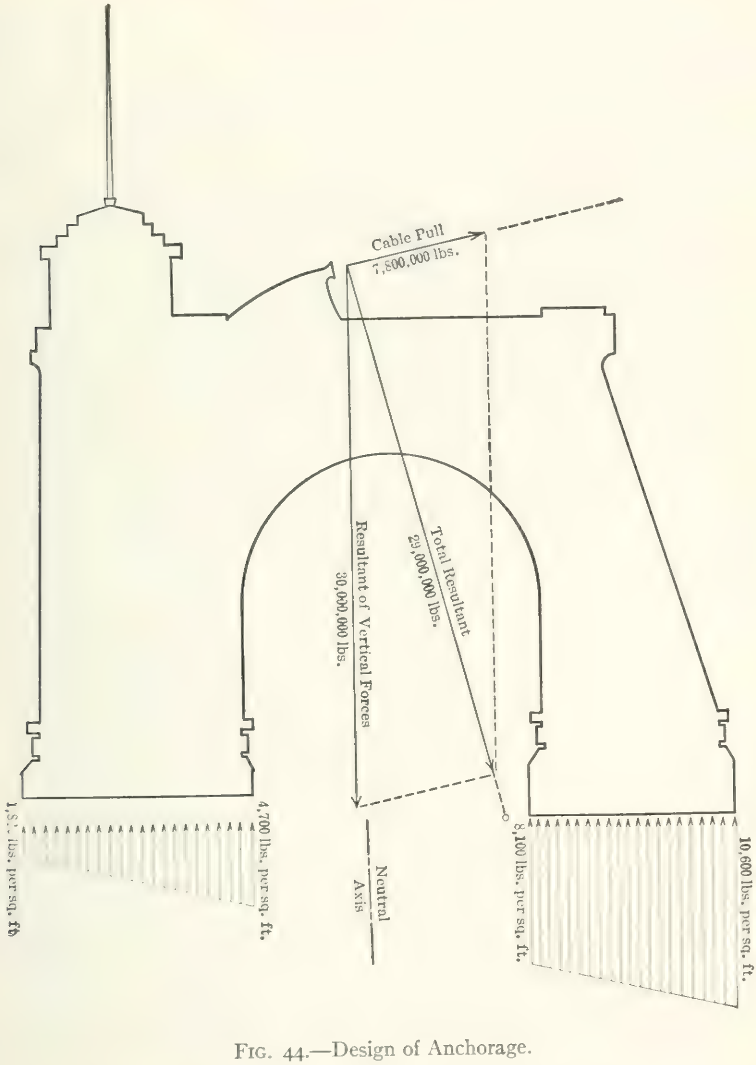


FIG. 44.—Design of Anchorage.

For the right side span, the shears will be the same, with the signs changed.

The temperature shears are given by Eq. (218):

$$V_t = -H_t \cdot \left(\tan \phi_1 - \tan \alpha_1 - \frac{cf}{l_1}\right) = -K \cdot H_t.$$

The shears are then multiplied by the respective secants of inclination, to obtain the stresses in the web members.

EXAMPLE 6

Design of Anchorage

1. Stability against Sliding.—The outline of a design for a reinforced concrete anchorage is shown in Fig. 44.

The principal forces acting are the cable pull, and the weight of the anchorage (including any superimposed loads). In the case at hand, the cable pull $= H \cdot \sec \phi = 7800$ kips. The weight of the anchorage and the superimposed loads is 30,000 kips. This weight is represented in the diagram as a vertical force drawn through the center of gravity of the anchorage and applied loads. By a parallelogram of forces, the total resultant is found, amounting to 29,000 kips. If its inclination from the vertical is less than the angle of friction, the anchorage is safe against failure by sliding.

2. Stability against Tilting.—The resultant is prolonged to intersection with the plane of the base, and its vertical component ($V = 28{,}000$ kips) is considered as an eccentric load applied at the point of intersection. The toe and heel pressures are given by,

$$p = \frac{V}{A} \pm \frac{Vec}{I},$$

where A is the area of the base (sq. ft.), I is its moment of inertia about the neutral axis (ft.^{4}), e is the distance (ft.) of the resultant V from the neutral axis, and c is the distance (ft.) from the neutral axis to the respective extreme fiber. We thus obtain, for the case at hand, a toe pressure of 10.6 kips per sq. ft. and a heel pressure of 1.8 kips per sq. ft. The allowable foundation pressure was 6 tons per sq. ft., so the anchorage figures safe against settlement or overturning.

CHAPTER IV

ERECTION OF SUSPENSION BRIDGES

1. Introduction.—The erection of suspension bridges is comparatively simple, and is free from dangers attending other types of long span construction.

The normal order of erection is: substructure, towers and anchorages, footbridges, cables, suspenders, stiffening truss and floor system, roadways, cable wrapping.

The cables are the only members requiring specialized knowledge for their erection. The other elements of the bridge, for the most part, are erected in accordance with the usual field methods for the corresponding elements of other structures.

2. Erection of the Towers.—The erection of the towers may proceed simultaneously with the construction of the anchorages.

In the case of the Manhattan Bridge, the tower (Fig. 45) consists of four columns supported on cast-steel pedestals resting on base plates set directly on the masonry pier. The sections of the pedestals (weighing up to 40 tons) were delivered by lighters and lifted by their derricks to the pier-tops; they were rolled into position on cast-steel balls placed on the bed plate, and then jacked up to release the balls.

The tower columns were erected by the use of ingenious derrick platforms (one for each pair of columns) adapted to travel vertically up the tower as the erection proceeded (Fig. 45). Each platform (21 feet by 34 feet) projected out from the face of the tower on the shore side and was supported by two bracket-struts below. The tipping moment was resisted by two pairs of rollers or wheels, one at each column, engaging vertical edges of the projecting middle portion of the column, the upper wheel being on the river side and the lower wheel on the shore side. The vertical support was furnished by hooks engaging the pro-

jecting gusset plates of the bracing system. A stiff-leg derrick with 45-foot steel boom was mounted at the middle of the inner side of the platform, being braced back to the outer corners of the platform. With this derrick the sections of the tower (weighing up to 62 tons) were lifted from the top of the pier and set in place, the material having been transferred from scows to the pier by floating derricks. When a full section had been added to the tower, blocks were fastened to the top and falls

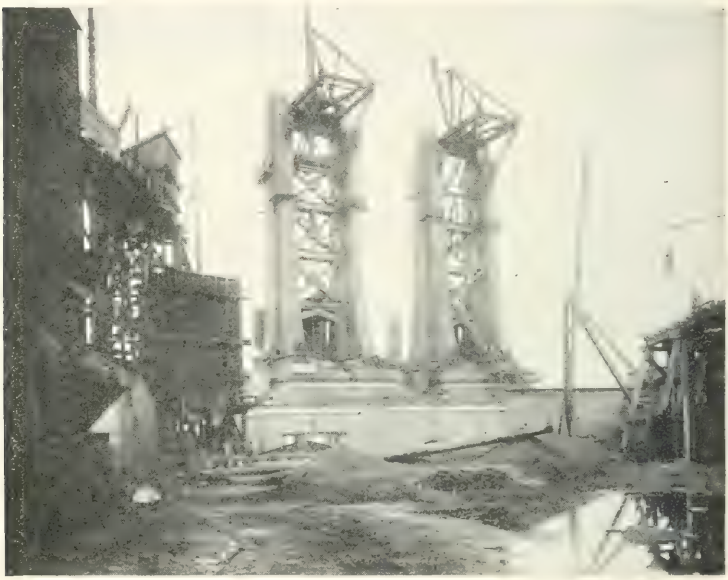


FIG. 45.—Manhattan Bridge. Erection of Towers.

(See Fig. 35, page 97).

attached to the derrick platform by which it then lifted itself to the next level.

For purposes of handling and erection, each column was divided by transverse and longitudinal field splice joints into sections of convenient size. The transverse joints were 12 feet to $27\frac{1}{2}$ feet apart, and were staggered to break joint. Where the three longitudinal sections changed to two, shim plates were used to level off. The riveting of the field splices (with 1-inch rivets) was kept several sections back of the erection work in

order to give opportunity for the transverse joints to come to full bearing.

Each tower column was finished with a cap section (52 tons) upon which was set the saddle (15 tons).

In addition to the two traveling derricks, the following equipment was required for the erection of each tower: two hoisting engines on the pier; one stiff-leg derrick (10-tons, 60-foot boom) on the pier between the tower legs, used in the assembly of the traveling derricks; two large storage scows moored to the pier, supplying the respective traveling derricks; a power plant on shore with two 50-H.P. horizontal boilers, a steam turbine blower for forced draft, and an air compressor; 30 pneumatic riveting hammers; 6 pneumatic forges.

The force at each tower consisted of a hundred men, including six riveting gangs. Riveting scaffolds were erected around the tower for field riveting, and were provided with stairs and safety railings. The erection record was 2000 tons of steel at one tower in sixteen working days.

Figure 46 shows the completed tower, 282 feet high above the masonry, and weighing 12,500,000 pounds.

For the Manhattan tower of the Williamsburg Bridge, a stationary derrick on the approach falsework was used to erect the steel up to roadway level; the erection was then completed by two stiff-leg derricks mounted on a timber tower built up on the cross-girder between the two tower legs. (The completed tower is shown in Fig. 57.)

For smaller bridges, the towers may be erected by gin-pole or by stationary derrick alongside. For the suspension bridge at Kingston, N. Y. (H. D. Robinson, Chief Engineer), a guyed derrick with 95-foot steel boom was set up on a square timber tower 80 feet high, for the erection of each steel tower; the same derricks later erected the adjoining panels of the stiffening truss (Fig. 56).

3. Stringing the Footbridge Cables.—The first step in cable erection consists in establishing a connection between the two banks. Various methods have been used since prehistoric times, when the first thread was fastened to an arrow and shot across

from bank to bank. In building the Niagara bridge, a kite was used to take the first string across the gorge; at other places, a light rope is drawn across with a rowboat.

In the erection of the Brooklyn Bridge, a $\frac{3}{4}$-inch wire rope was first laid across the East River by means of a tugboat and scow, and then raised to position. With another line taken over in the same manner, an endless rope was made, and this was used for hauling over the remaining traveler ropes and an

Fig. 46.—Manhattan Bridge. Erection of Footbridges.

auxiliary $1\frac{3}{4}$-inch carrier rope; the latter served to carry the load of the footbridge cables and cradle cables ($2\frac{3}{8}$ and $2\frac{5}{8}$ inches diameter) when these were hauled across the river.

For the Manhattan Bridge, sixteen $1\frac{3}{4}$-inch wire ropes were swung between the towers in four groups of four (Fig. 46). One group (to make a single footbridge cable) was taken across at a time. The four reels were mounted on a scow brought alongside one of the towers, *A*. The end of each rope was unreeled and hauled up over a temporary cast-iron roller saddle mounted

on top of the tower, and thence carried back to the anchorage *A*, where it was made fast. Then the scow was towed across the river, laying the ropes along the bottom, to the opposite tower *B*. The remainder of each rope was then unreeled and coiled on the deck of the scow. Then, while all river traffic was stopped for a few minutes, the free end of each rope was hauled up by a line to the top of Tower *B*, over a roller saddle and thence to the Anchorage *B*; the middle of the rope, or bight, rose out of the water during this operation, and came up to the desired position. After being made fast at the Anchorage *B*, the ropes were socketed and drawn up to the precise deflection desired, as determined by levels. Each group of four ropes then formed a temporary cable for the support of the footbridges (Fig. 46).

The footbridge ropes for the Williamsburg Bridge were strung in the same manner as for the Manhattan Bridge, except that three were laid, instead of four, at each trip of the flatboat across the river.

4. Erection of Footbridges.—The next step is the construction, for each cable, of a footbridge or working platform which permits the wires to be observed and regulated throughout their length and greatly facilitates the entire work on the cables.

For the Manhattan Bridge (Fig. 46), traveling cages, hanging from the footbridge cables as a track, were used by the men placing the double cantilever floorbeams. These floorbeams, $35\frac{1}{2}$ feet long and spaced 21 feet apart, were supported on the footbridge cables in pairs, and were secured to the upper side of these cables by U-bolt clamps. Upon the outer portions of the floorbeams were dapped the stringers, three lines on each side, and on these were spiked the floorboards, spaced $1\frac{1}{2}$ inches apart in the clear. In this manner, four platforms were constructed, 8 feet wide, placed concentric with the main cables and 30 inches (clear) below them. The platforms were provided with wire-rope handrails. Passage from one platform to another was provided only at the towers and anchorages, and at mid-span. Each platform carried nine small towers called "hauling towers" (Fig. 47), about 250 feet apart, to support the sheaves of the carrying and hauling ropes used for placing the strand

wires. The platforms were braced and guyed underneath by backstays from each tower, and by inverted storm cables connected to them at intervals of 54 feet. The entire construction was of light wooden plank (maximum size 3×12) and all connections were thoroughly bolted with washer bearings. All woodwork had been previously cut to length, framed, bored and marked, and hoisted to the tops of the towers. The floorbeams were slipped down on the cables toward the center of the span

FIG. 47.—Manhattan Bridge. Footbridges and Sheave Towers.

and toward the anchorages, set by the men in the traveling cages, and maintained in position by the stringers dapped to them. Then the sheave towers and handrails were erected on the platforms, practically completing the falsework (Fig. 47).

The temporary platforms for the Williamsburg Bridge are shown in Fig. 57. Two footbridges were used, 67 feet center to center, connected by transverse bridges every 160 feet.

For the Brooklyn Bridge, Fig. 25, the timber staging consisted of one longitudinal footbridge and five transverse plat-

forms, called "cradles," from which the wires were handled and regulated during cable-spinning.

5. Parallel Wire Cables. Smaller spans have been built with ready-made parallel wire cables, served with wire wrapping at short intervals; but the individual wires in such cables lack freedom to adjust themselves to the necessary curvature of suspension, so that objectionable stress conditions arise. For these reasons it has become general practice to use the method, introduced by Roebling, of spinning the desired number of parallel wires in place and then combining them into a cable. The cable is pressed into cylindrical form and wound with continuous wire wrapping. This wrapping, together with the tight cable bands to which the suspenders are attached, serves to create enough friction pressure between the wires to ensure united stress action.

Guide wires are used as a means of adjusting the individual wires to equal length. Slight differences in length, if distributed over the entire span, will be immaterial. To avoid the excess in length of the longer wires from accumulating at a single point, the wire wrapping should be started at a considerable number of points distributed along the cable; and a large cable should first be bound into smaller temporary strands by serving with wire at intervals.

The length of the guide wire must be accurately computed, so that the resulting cable shall have the desired sag (assumed in the design) after the bridge is completed. Length corrections must be made for any cradling of the cables, variation from mean temperature, the curve of the cable saddle, and the elastic elongation due to the suspended load.

6. Initial Erection Adjustments.—Special computations have to be made for the location of the guide wires, for setting the saddles on top of the towers, and for the length of the strand legs.

When the desired final position of the cable, under full dead load, is known, its length is carefully computed, including the main span parabola between points of tangency at the saddles, the short curved portions in the saddles, and the side-span parabolas or tangents from point of tangency at the saddle to

the center of shoe pin at the anchorage. Applying corrections for elastic elongation (due to suspended load) and for difference of temperature from the assumed mean, the length of unloaded cable is determined. This gives the length of the guide wire between the same points.

Assuming no slipping of the strands in the saddles, the initial position of the saddles is computed so as to balance tensions (or values of y) between the main and side span catenaries. This gives the distance the saddles must be set back (toward shore) from their final position on the tops of the towers.

Since the strands will be spun about 2 feet above their final position in the tower saddles, the initial position of the strand shoes will be a short distance forward of their final position. This distance is carefully computed and gives the required length of the "strand legs" (Fig. 48). The distance may also be determined or checked by actual trial with the guide wire.

Taking into consideration the previously calculated and corrected total length of cable between strand shoes, the initial raised position of the strands above the tower saddles, and the length of strand legs shifting the initial position of the strand shoes, the ordinates of the initial catenaries in main and side spans are carefully computed. These ordinates are used for setting the guide wires with the aid of level and transit stationed at towers and anchorages.

For the Cumberland River footbridge (540-foot span, see page 184), the saddles on the two towers were set back about 5 inches toward shore from center of tower. This distance was figured from backstay elongation and tower shortening due to dead load plus one-half live load, so that the center of the tower would bisect the movement of the shoe (on rollers) for live load at mean temperature. Allowing for displacement of saddles and cable stretch, the no-load cable-sag was made $38\frac{1}{2}$ feet in order that the sag in final position under full live load should be 45 feet.

In the case of the Brooklyn Bridge, the strands were spun about 57 feet above their final position at mid-span, the purpose, as stated, being to avoid interference with regulation and to increase the tension as an initial strength-test of the individual

wires. In consequence, the strand leg had to be designed so as to hold the shoe 12 feet *behind* the anchor pin. After the strand was finished, the shoes were let forward into their final places and, at the same time, the strand was lowered from the rollers on top of the saddle into the saddle, which double operation caused the vertex to sink into correct position as previously calculated.

For the Williamsburg Bridge, the strands were spun 15 feet above their final position, requiring the shoes to be initially set *back* of the anchor pin, as in the Brooklyn Bridge.

For the Manhattan and Kingston Bridges (H. D. Robinson, Engineer-in-charge), the strands were spun parallel to (and slightly above) their final position. In these cases, the strand leg held the shoe a short distance *in front* of the anchor pin; and the shoe had to be pulled back that distance when the strands were lowered into the saddle (Fig. 49).

In the case of the Kingston Bridge (705-foot span, Fig. 56), instead of setting the saddles back on the towers, the tops of the towers were tipped back toward the shore a distance of 6 inches, by means of temporary backstays, the anchor bolts at the toe being loosened ½ inch to permit the tilting. The cables were erected with the towers and the attached saddles in this position. As the steelwork in the main span was erected, the backstays gradually elongated until the towers returned to their final vertical position.

The initial erection adjustments for the Brooklyn, Williamsburg, Manhattan and Kingston Bridges are summarized and compared in the following table:

INITIAL POSITION OF CABLE STRANDS

(With Reference to Final Position)

	Height Above Crown	Height Above Saddle	Distance Saddle Set Back	Distance Shoe Set Forward
Brooklyn	57 ft.	2.1 ft.	0.1 ft.	−12 ft.
Williamsburg	15	2	2.75	− 3
Manhattan	2	2	0	+ 1.83
Kingston	1.25	1.25	0.5	+ 0.25

7. Spinning of Cables.—The operation of cable-spinning requires an endless wire rope or "traveling rope" (Fig. 48) suspended across the river and driven back and forth by machinery

FIG. 48.—Strand Shoes and Traveling Sheaves Ready for Cable Spinning. (Manhattan Bridge.)

for the purpose of drawing the individual wires for the cable from one anchorage to the other. There is also suspended a "guide wire" which is established by computations and regu-

lated by instrumental observations so as to give the desired deflection of the cable wires.

Large reels, upon which the wires are wound, are placed at the ends of the bridge alongside the anchor chains (Fig. 48). The free end of a wire is fastened around a grooved casting of horseshoe outline called a "shoe" (Fig. 48), and the loop or bight, thus formed is hung around a light grooved wheel (Fig. 48) which is fastened to the traveling rope. The traveling rope with its attached wheel, moving toward the other end of the bridge, thus draws two parts of the wire simultaneously across from one anchorage to the other; one of these parts, having its end fixed to the shoe, is called the "standing wire"; while the other, having its end on the reel, is called the "running wire" and moves forward with twice the speed of the traveling rope. Arriving at the other end, the wire loop is taken off the wheel and laid around the shoe at that end. The two parts of the wire are then adjusted so as to be accurately parallel to the guide wire, the operation of adjustment being controlled by signals from men stationed along the footbridge. The wire is then temporarily secured around the shoe, and a new loop hung on the traveling wheel for its second trip. After two or three hundred wires have thus been drawn across the river and accurately set, they are tied together at intervals to form a cable strand.

For the Manhattan Bridge, the wires (drawn in 3000-foot lengths) were spliced to make a continuous length of 80,000 feet (4 tons) wound on a wooden reel (Fig. 48). These reels were 48 inches in diameter (at bottom of groove) and 26 inches long, and were provided with brake drums. On each anchorage were set eight reel stands, each with a capacity of four reels.

The equipment used for cable-spinning consisted of an endless $\frac{3}{4}$-inch steel traveling rope passing around a 6-foot horizontal sheave at each anchorage; machinery for operating the endless rope; devices for removing and adjusting the wires and strands; apparatus for compacting and wrapping the cables; hoisting machinery and power plant.

Attached to the endless rope ("traveling rope") at two

equidistant points, were deeply-grooved 4-foot carrier sheaves ("traveling wheels") in goose-neck frames (Fig. 48). These frames were held securely in a vertical plane, and were designed with clearance to ride over the supporting sheaves.

The "strand shoe" was held 22 inches in front of final position by a special steel construction called a "strand leg" (Fig. 48) attached to the pin between two anchorage eyebars.

The bight of wire was placed around the traveling wheel and

Fig. 49.—Manhattan Bridge. Anchoring a Completed Strand.

pulled across. As each part of the wire became dead, it was taken by an automatic Buffalo grip at the tower and, with a 4-part handtackle of manila rope, adjusted to the guide wire. It required about seven minutes for a trip across from anchorage to anchorage (3223 feet). Only ten field splices were required to a strand (256 wires). After the strand was completed, the wires were compacted with curved-jaw tongs and fastened (or "seized") with a few turns of wire, every 10 feet. Then, with a "strand-bridle" attached to a 35-ton hydraulic jack (Fig. 49),

the shoe was pulled toward the shore, releasing the strand leg and the eyebar pin. The strand shoe was then revolved 90° to a vertical position (Fig. 49), and pulled back to position on the eyebar pin.

The strand was then lifted from the temporary sheaves in which it was laid at the anchorages and the towers, and lowered into the permanent saddles; a 20-ton chain hoist and steel "balance beam" were used for this operation. The strand was then adjusted to the exact deflection desired, by means of shims in the strand shoe.

After the seven center strands were completed, they were bunched together with powerful squeezers to make a cylinder about $9\frac{1}{4}$ inches in diameter, secured with wire "seizings" at intervals. Then the remaining strands were completed, and compacted in two successive layers around the core the interstices being filled with petrolatum. A hydraulic compacting machine was used for this squeezing, and temporary clamps applied.

Then the cable was coated with red-lead paste, and the permanent cable bands and suspenders were attached.

After the stiffening trusses and floor were suspended, the spaces between the cable bands were covered with wire wrapping.

The spinning of these cables took six days for a strand (256 wires); but four strands in each cable were strung simultaneously. The four cables (each consisting of thirty-seven strands or 9472 wires) were completed in four months. The work of compressing and binding the cables and attaching the suspender clamps and ropes took two or three months more, but the erection of the suspended trusses proceeded at the same time.

As soon as the strands were completed, the footbridges were hung to the main cables to be later used for the work of cable wrapping. The temporary footbridge cables were cut up for use as suspenders.

For the Williamsburg Bridge (Fig. 57), the wire was supplied on 7-foot wooden reels carrying 90,000 feet (9000 pounds) per reel. An engine on the New York side operated the driving

wheels around which two endless ropes passed. Two carrier sheaves on each endless rope traveled back and forth, carrying two bights across (for two strands) on the forward trip, and two bights (for two adjacent strands) on the return trip. In this manner each endless rope was laying four strands at the rate of fifty wires in each strand in ten hours.

Eight reels of wire were required for each strand. When the end of a coil was reached, it was held in a vise and connected to a wire from a fresh reel, by screwing up a sleeve nut over the screw-threaded ends (which were formed by a special machine to roll the threads).

As the wire was laid, it was adjusted to conform to the catenary of the guide wire, in order to secure uniform tension in the wires of the finished cable.

The carrier wheels moved 400 feet per minute. There were three men at each anchorage to handle the reels, make splices, adjust the wire and take the bights off and on the carrier wheels. As the carrier wheel passed each tower, three men on the top of the tower clamped handtackle to the wire and pulled up until the wire was adjusted exactly parallel to the guide wire, as signaled by men distributed along the footbridge (three men on each side span and seven men on the main span). These men clamped the wire to the strand after adjustment. After the standing wire was adjusted, the running wire was regulated in the same manner, but in the reverse order. A total of twenty-five men were thus required to handle the wire as it was laid.

As soon as the strand was completed, the shoe was drawn clear of its support by a 25-ton ratchet jack anchored to the masonry. Then the shoe was twisted by hand with a long bar and thus revolved 90° into a vertical plane and allowed to slip back towards the tower, thus lowering the strand in the middle of the main span. The shoe was then permanently connected to the end pin of the anchor-chain eyebars. Shims back of the pin in the slotted pin-hole of the strand shoe provided adjustment for the strand length; each $\frac{1}{8}$-inch shim corresponded to a vertical movement of about 1 inch at mid-span.

When the inner strands were completed, their ties were

removed and they were made into one strand to avoid trouble in handling them after they were surrounded by the remaining strands.

8. Compacting the Cables.—Each cable consists of 3, 7, 19 or 37 strands, depending upon its size, and these have to be compacted to make a cylindrical cable.

For the Manhattan Bridge, the temporary seizings around the strands were removed and the cable was compacted by hydraulic squeezers. Sixteen duplicate squeezers were used, each consisting of a hinged collar with a hydraulic jack of 6-inch stroke opposite the hinge. A hydraulic hand-pressure pump was used to produce a pressure of 5000 pounds per square inch or a total force of 43,000 pounds on the squeezer piston. Seizing (12 turns of No. 8 wire) was applied close to the squeezer, which was then moved 2 feet forward to repeat the operation. With two men operating each squeezer, the four cables were compacted in a few weeks.

9. Placing Cable Bands and Suspenders.—After the cables are compacted (with wire seizing at short intervals to hold them), the cable bands are placed at the panel points.

For the Manhattan Bridge, the cable bands (Fig. 55) consist of split cast-steel sleeves, 3 feet long, with ten $1\frac{5}{8}$-inch bolts through the longitudinal flanges. The upper half has two semicircular grooves, 12 inches apart, for holding the suspender ropes. The bolts were screwed up tight by means of socket wrenches with 4-foot handles, operated by two or three men each.

The $1\frac{3}{4}$-inch suspender ropes, made by cutting up the temporary footbridge cables, were fitted with cast-steel sockets $5\frac{1}{4}$ inches in diameter by 17 inches long. These sockets were threaded on the outside to receive a cast-steel nut $5\frac{1}{4}$ inches thick. The ends of the rope were served; and the wires beyond the serving were spread, cleaned in dilute acid, washed in water and dried with a painter's torch. The end of the rope was then passed through the socket (which had been carefully cleaned of sand and scale), the wires were spread to fill the covered portion, and melted spelter (heated to a very thin consistency) was

poured in, filling all the interstices. Some of the finished ropes were tested, and showed an ultimate strength of 287,000 to 290,000 pounds, with the rope breaking 4 to 8 feet from the socket; there was no sign of injury at the socket, thread or nut.

The suspenders were then placed in position around the cable bands, with their lower ends ready to engage the bottom chords of the stiffening trusses (Fig. 50).

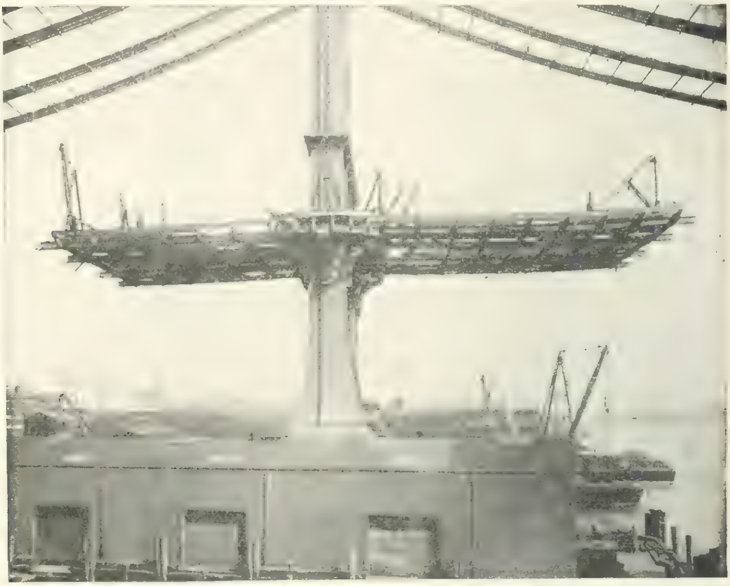


FIG. 50.—Manhattan Bridge. Erection of Lower Chords and Floor System.

10. Erection of Trusses and Floor System.—The suspension from the cables permits the steelwork to be erected without falsework. In planning the program of erection there must be considered the method of connection to the suspenders, clearances for travelers, and the reach of the booms. In addition, the scheme should aim to balance the dead-load distribution along the span, so as to minimize the distortion of the cables during erection.

In the Manhattan Bridge, the truss is supported at each

panel point by four parts of $1\frac{3}{4}$-inch steel rope suspenders (Fig. 50) with their bights engaging the main cables and having, at the lower end, nut bearings on horizontal plates across the bottom flanges of the lower chord.

All members were shipped separately, the chord members in two-panel-length pieces weighing 26,000 to 30,000 pounds each.

The erection proceeded at four points simultaneously, working in both directions from each tower (Fig. 50). Traveler der-

FIG. 51.—Manhattan Bridge. Erection of Verticals.

ricks of 25-ton capacity were used, with 34-foot mast and 50-foot boom (covering two panels in advance) and provided with bull-wheel. At each point of erection there were two of these large derricks, also one jinnywink derrick with 30-foot boom and 7-ton capacity. In addition to these twelve movable derricks, there were four stationary steel-boom derricks at the towers.

Starting at the towers, the lower chords and floor system were assembled two panels in advance of the travelers, making temporary connections to the suspenders, until the anchorages

and mid-span were reached. Then the travelers returned to the towers to commence their second trip.

The material was hoisted by the tower derricks and loaded on service cars which delivered it to the traveler derricks. The service cars ran on the permanent track between the inner and outer trusses; the cars were hauled away from the tower by cables operated by hoisting engines on the tower, and returned empty, by gravity, on the grade furnished by the camber.

FIG. 52.—Manhattan Bridge. Erection of Diagonals.

On the first trip, the lower chords, lower deck and verticals were erected (Fig. 51); on the second trip, the truss diagonals were erected (Figs. 52, 53); and on the return (Fig. 54), the upper deck and transverse bracing were put up, thus completing the structure.

On the first trip, temporary suspender connections were made to the lower chord at alternate panel points so as to miss the upper chord splices. After the return of the travelers, permanent connection and adjustment of the suspenders at the other points

were made. The temporary suspender connections were removed before the top chords were erected, and were connected again (permanently) after the top chords were in place.

A force of three hundred men was employed on this work, and their record was 300 tons of steel erected in a day. There were about 1,000,000 field rivets in the three spans. The bridge was formally opened ten months after the floor hanging commenced.

Where the side spans are not suspended from the cables,

FIG. 53.—Manhattan Bridge. View before Erection of Top Chords.

falsework is generally required. In the Kingston Suspension Bridge, Fig. 56, the side spans (although suspended) were erected on light falsework, as time was thereby saved.

The first few panels of the main span are generally erected by the stationary derricks at the tower, as far as their booms can reach. Additional panels may be erected by drifting or outhauling from the cable; or by the use of "runners," that is, block and falls suspended from the advance cable band and

operated by the hoisting engine at the tower. At Kingston, the latter method was adopted, dispensing with the use of travelers for the main portion of the span.

11. Final Erection Adjustments.—The equilibrium polygon is computed for the dead load acting on the cable, and levels taken at a number of points on the cable should check these ordinates. The elevations and camber of the roadway are also

Fig. 54.—Manhattan Bridge. Erection of Top Chords.

checked with levels and corrected, where necessary, by adjusting the lengths of the suspenders.

In completing the stiffening truss, the closing chord members should be inserted after all the dead load is on the structure, the connecting holes at one end being drilled in the field.

If the closure of the stiffening truss has to be made before full dead load is on the structure, or at other than mean temperature, the vertical deflections are computed for these variations from assumed normal conditions and the suspenders adjusted accordingly, before connecting the closing members.

In adjusting the suspenders, the center hanger is shortened or lengthened the calculated amount, and the other hangers are corrected by amounts varying as the ordinates to a parabola.

If the trusses are assembled on the ground before erection, the exact camber ordinates can be measured and reproduced (by suspender adjustment), so as to secure zero stress under full dead load at mean temperature.

An ideal method of checking the final adjustments is by means of an extensometer, which should check zero stresses throughout the stiffening truss when normal conditions are attained, or calculated stresses for any variation from assumed normal conditions.

Instead of adjusting to zero stress for full dead load, it would be more scientific and somewhat more economical to adjust for zero stress at dead load plus one-half live load.

12. Cable Wrapping.—Close wire wrapping has proved to be the most effective protection for cables.

For the Manhattan Bridge, No. 9 galvanized soft-steel wire (0.148-inch diameter) was used. This was rapidly wound around the cable by a very simple and ingenious self-propelling machine operated by an electric motor. This machine, designed by H. D. Robinson, is illustrated in Fig. 55.

In advance of the machine, the temporary seizings are carefully removed and the cable painted with a stiff coat of red-lead paste. The end of the wrapping wire is fastened in a hole in the groove at the end of the cable band. The machine, carrying the wire on two bobbins or spools, travels around the cable and applies the wire under a constant tension. The machine presses the wire against the preceding coil and at the same time pushes itself along. The rate is about 18 feet per hour.

The machine weighs 1000 pounds and is operated by a $1\frac{1}{2}$-H.P. motor at a speed of 13 R.P.M. It is handled by a force of six men.

(The small hand-operated device, which was superseded by the motor-driven machine, is seen at the extreme right in Fig. 55. It was used to complete the wrapping close to the cable bands.)

For the Williamsburg Bridge, wire wrapping was not used; instead, the cables were covered with a preservative coating of oil and graphite, then wrapped spirally with three layers of waterproof duck, and finally enclosed in a thin steel-plate shell made in two semi-cylindrical portions with overlapping joints and locked fastenings. This protection has proved inadequate to keep out moisture and prevent rust, and it has recently (1917–1921) been replaced by wire wrapping applied with Robinson's machine.

FIG. 55.—Manhattan Bridge. Cable Wrapping Machine.

13. Erection of Wire Rope Cables.—The individual wire ropes composing a cable of this type may be towed across the river in the same manner as the temporary footbridge ropes of a parallel wire cable; or they may be strung across by means of a single working cable stretched from tower to tower.

The latter method was used for a footbridge of 540-foot span built in 1919 over the Cumberland River by the American Bridge Co. Each cable consisted of seven ropes of $1\frac{7}{8}$-inch diameter.

A working cable of 1-inch wire rope was first stretched across between the towers for each of the main cables. The main ropes were unwound from the reels back of one tower. One end of a rope was lifted to the top of the tower and hauled across the river to the top of the opposite tower, the rope being supported from the 1-inch working cable by blocks attached at intervals of about 60 feet, thus preventing too much sag. The

FIG. 56.—Erection of Rondout Creek Bridge at Kingston, N. Y., 1921. Type 0*S*.

Span 705 feet.

rope was then lowered to approximately correct position, and the sockets attached to the tower shoes. The remaining ropes were then stretched in the same manner, and all were then adjusted by nuts at the ends until they touched a level straightedge held on the fixed line of sag determined by a transit in the tower. The cable clamps and suspenders were then placed by men on a movable working platform hung from the cables, beginning in the center and working toward each end. A "boatswain chair" was used to carry out men, materials and

tools. The floor system was also erected by men on the working platform, in this case working from both ends toward the center. The platform was then removed, and the trusses were erected from the ends toward the center by workmen on the floor system, using the two working cables (shifted to the center of the bridge) as a trolley cable for transporting the truss sections to position. When the top lateral bracing, railings, and wood floor were added, the structure was completed. A total

FIG. 57.—Footbridges for Erection of Williamsburg Bridge.

(See Fig. 31, page 88).

of 205 tons of structural steel and 45 tons of cables were thus erected in a period of twelve weeks.

14. Erection of Eyebar Chain Bridges.—Chain suspension bridges have, as a rule, been erected upon falsework.

The falsework used for the erection of the Elizabeth Bridge at Budapest (1902, Span 951 feet) is shown in Fig. 58. The falsework consisted of huge scaffoldings built on piles and protected from floating ice by ice breakers. Four openings of 160 feet were left for vessels; these openings were spanned by

temporary timber bridges floated into place on pontoons. After the falsework was completed, the main chains were erected in twelve weeks. The falsework was then taken down and the steelwork completed.

At a crossing like the East River or the Hudson River, the use of such falsework would be out of the question. A comparison of the cumbersome construction employed for the Elizabeth Bridge (Fig. 58) with the comparatively insignificant scaffolding required for the Williamsburg Bridge (Fig. 57), is an argument for wire cable *vs.* eyebar bridges.

FIG. 58.—Falsework for the Elizabeth Bridge (Eyebar Chains).

(See Fig. 34, page 95).

A different scheme, eliminating heavy falsework, was used for the Clifton Bridge (1864, Span 702 feet). Under each set of three chains, a suspension footbridge was constructed, using wire ropes. Above this staging, another rope was suspended to carry the trolley frames for transporting the links. The chains were commenced simultaneously at the two anchor plates, the lowest of the three chains being put in first. Commencing at the anchorage, there were inserted the whole number of links, namely 12, then 11, 10, 9, 8, and so on until the chain was

diminished to 1 link; then the chain was continued with 1 and 2 links, alternately, until the two halves met at mid-span. The suspended footbridge was strong enough to carry the weight of this chain (consisting of 1 and 2 links, alternately) until the center connection was made; the chain was then made to take its own weight by removing the blocking under it. The next operation was to add the remaining links of the chain on the pins already in place. The process was repeated for the upper chains, and then the roadway was suspended.

The Cologne Suspension Bridge (1915, Span 605 feet, Fig. 17), was the first large bridge to be built hingeless. (The Kingston Bridge, 1921, was the second.) It is of the self-anchored type, the stiffening girder taking up the horizontal tension; and the towers are hinged at the base. Nickel steel was used for the chains, the eyebars being of the European type, that is, of flat plates (36 to 59 inches wide) bored for 12-inch pin-holes near the ends. The erection of the chains and stiffening girders proceeded simultaneously on special staging, and was so conducted that the girders were completed first. The girders were made three-hinged during erection and then changed to hingeless by riveting on splice plates.

The procedure was as follows: Falsework was built for the side spans and a traveler was assembled at each end. The side-span girders and deck were erected on the falsework, and the staging built up (on the girders) for the land chains, the traveler moving forward from the anchorage to the tower during this operation. The traveler then moved out on cantilever false-work spans in the main opening, erecting the stiffening girders and the staging for the chains from the tower to mid-span. The erection of the chains followed closely upon the erection of the girders. When the stiffening girders were completed and the suspension chains connected to the ends (with 24-inch pins), every third hanger was coupled up. The staging carrying the chains was then removed, and the remaining hangers were connected and adjusted by means of their turnbuckles to bring the pin points in the chains into their correct positions. The splices in the webs and flanges of the stiffening girders at the

three hinge-points were then riveted up, thus completing the erection.

For Lindenthal's Quebec Design (1910, Span 1758 feet, Fig. 40), the following scheme of erection was proposed: The side spans were to be erected on steel falsework—first the floor system, then the eyebars and pins of the lower chord chain, then the verticals and upper chain eyebars, leaving the pins projecting out to receive the diagonals and remaining eyebars after the main span chains were erected and self-supporting. The towers were to be riveted up in place and temporarily anchored to the steel staging which, in turn, was to be anchored to the abutment. The first sets of eyebars (one and two alternating per panel) of the chains of the middle span were to be erected from temporary wire rope cables, each consisting of forty steel wire ropes of $2\frac{1}{2}$-inch diameter. Then the remaining eyebars and gusset plates were to be pushed on to the pins until the chains were completed. Thereafter the verticals and diagonals were slipped in place, and the suspenders and floor system of the middle span erected.

15. Time Required for Erection.—The time schedule for the Manhattan Bridge (1470-foot span, Fig. 35) was as follows:

First substructure contracts let		1901
Pier foundations commenced	May,	1901
Work commenced on final (revised) design	March,	1904
Steel towers commenced	July,	1907
Steel towers completed (12,500 tons)	July,	1908
Temporary cables strung	June 15–20,	1908
Footbridges constructed	July 7–13,	1908
Spinning of main cables commenced (4 cables)	Aug. 10,	1908
Last wire strung (37,888 wires)	Dec. 10,	1908
Erection of suspended steel commenced	Feb. 23,	1909
Erection of suspended steel completed (24,000 tons)	June 1,	1909
Approaches completed and bridge formally opened	Dec. 31,	1909

The steel erection, amounting to 42,000 tons of steel between anchorages and including towers, cables, trusses and decks, was accomplished in two and a half years.

The Kingston Suspension Bridge (705-foot span, Fig. 56) was completed in one year (1920–1921), although several

months were lost in waiting for steel delivery. The bridge contains 1600 tons of structural steel and 250 tons of cables.

The 400-foot-span suspension bridge at Massena, New York, (Fig. 30; H. D. Robinson, Consulting Engineer) containing 400 tons of steel, was erected complete in six months.

APPENDIX

DESIGN CHARTS FOR SUSPENSION BRIDGES

INTRODUCTION.—To expedite the proportioning or checking of suspension bridges, the author has devised the three charts which are presented in this Appendix. These charts give directly the maximum and minimum moments and shears in the stiffening truss, throughout the main and side spans. The charts are constructed for the usual form of construction, parabolic cable with two-hinged stiffening truss; and they cover both types:

Type $2F$—Free Side Spans (Straight Backstays).

Type $2S$—Suspended Side Spans (Curved Backstays).

To use the charts, it is simply necessary to calculate N, which is a constant for any given structure. This constant N is defined by Eq. (125) or (167), Chapter I; the formulas for N are also reproduced on the charts. In these formulas:

I = moment of inertia of the truss, main span;
I_1 = moment of inertia of the truss, side span;
A = area of cable section, main span;
A_1 = area of cable section, side span;
E = coefficient of elasticity for truss;
E_c = coefficient of elasticity for cable;
f = cable sag, main span;
f_1 = cable sag, side span;
l = main span of cable (c. to c. of towers);
l' = main span of truss (c. to c. of bearings);
l_1 = side span of truss (c. to c. of bearings);
l_2 = side span of cable (tower to anchorage);
α_1 = inclination of cable chord in side span.

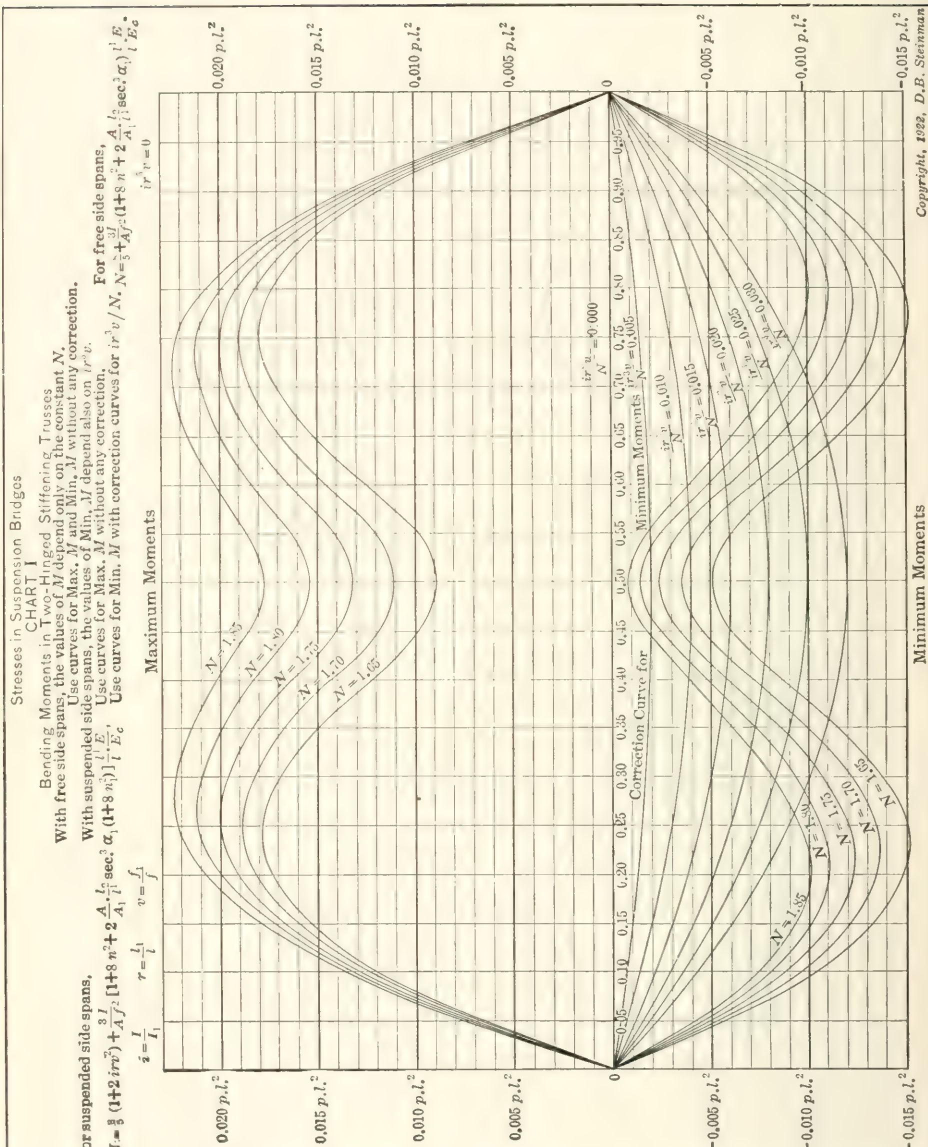


CHART I.—Bending Moments in Main Span.

The value of N is usually about 1.70 for the case of free side spans (Type $2F$), and about 1.80 for the case of suspended side spans (Type $2S$).

For the case of suspended side spans (Type $2S$) it is also necessary to figure the ratio-product ir^3v, where

$$i=\frac{I}{I_1}, \quad r=\frac{l_1}{l}, \quad v=\frac{f_1}{f}.$$

This ratio-product is also a constant for any given structure. (It is equal to zero when the backstays are straight, Type $2F$. For Type $2S$ we may usually assume $i=1$, and $v=r^2$, so that $ir^3v=r^5$, approximately.)

With the values of the two constants N and ir^3v known, the maximum and minimum moments and shears for all points in main and side spans may be read directly from the charts, thus dispensing with the usual laborious computations.

Chart I.—Bending Moments in Main Span.—This chart gives the governing bending moments throughout the main span. The upper curves (for different values of N) give the maximum bending moments, and the lower curves (for different values of N) give the minimum bending moments. No correction is required except for minimum moments in the case of suspended side spans (Type $2S$). The corrections for this case are given by the parabolic curves plotted below the axis $\left(\text{for different values of } \dfrac{ir^3v}{N}\right)$. These corrections, like the minimum moments, are negative in sign, and the two should therefore be added arithmetically. (These corrections represent the effect of load covering both side spans.)

The values of Total M (for full loading of all spans) may be obtained, if desired, by arithmetically subtracting the corrected Min. M from Max. M. Total M for load covering the main span alone may be obtained by subtracting the uncorrected Min. M from Max. M. The resulting values, in either case, would be represented by parabolas above the axis.

Chart II.—Shears in Main Span.—This chart gives the governing shears throughout the main span. The upper curves (for

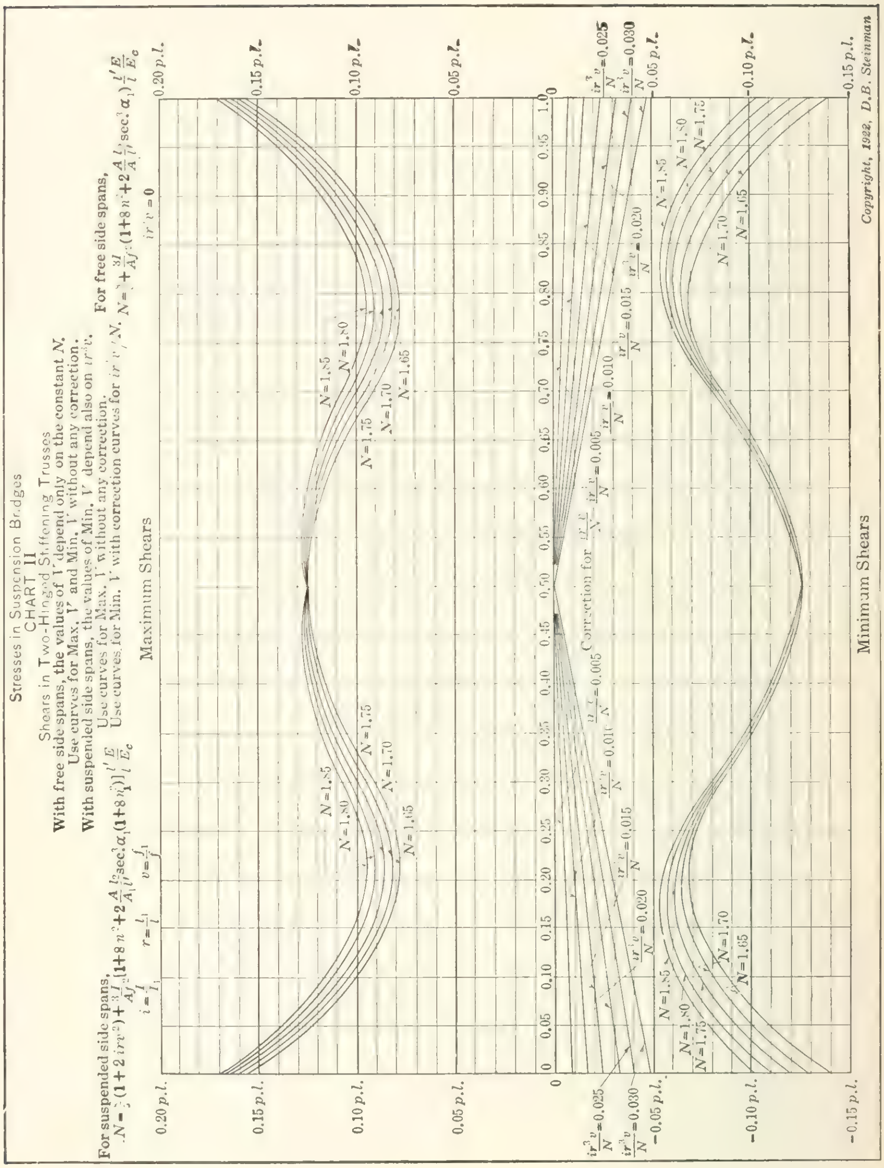


CHART II.—Shears in Main Span.

different values of N) give the maximum shears, and the lower curves (for different values of N) give the minimum shears. No correction is required except for minimum shears in the case of suspended side spans (Type 2S). The corrections for this case are given by the straight lines plotted below the axis $\left(\text{for different values of } \frac{ir^3v}{N}\right)$. These corrections are of the same algebraic sign as the minimum shears, and the two should therefore be added arithmetically. (These corrections represent the effect of load covering both side spans.)

On this chart, the plus sign indicates a shear upward on the outer side and downward on the inner side of a section; the minus sign indicates a shear in the opposite direction.

The values of Total V (for full loading of all spans) may be obtained, if desired, by arithmetically subtracting the corrected Min. V from Max. V. Total V for load covering the main span alone may be obtained by subtracting the uncorrected Min. V from Max. V. The resulting values, in either case, would be represented by radiating straight lines above the axis.

Chart III.—Moments and Shears in Side Spans.—This chart gives the governing stresses throughout a side span.

In the left-hand diagram, the upper parabolic curves $\left(\text{for different values of } \frac{ir^3v}{N}\right)$ give the maximum bending moments. (These curves represent the effect of load covering the given side span.) The lower parabolic curves $\left(\text{for different values of } \frac{1+ir^3v}{N}\right)$ give the minimum bending moments. (These curves represent the effect of load covering the two other spans.) The values of Total M (for load covering all three spans) may be obtained, if desired, by arithmetically subtracting Min. M from Max. M. The resulting values would be represented by flat parabolas above the axis.

In the right-hand diagram, the upper curves $\left(\text{for different values of } \frac{ir^3v}{N}\right)$ give the maximum shears. (These curves represent the effect of load covering the given side-span.) The

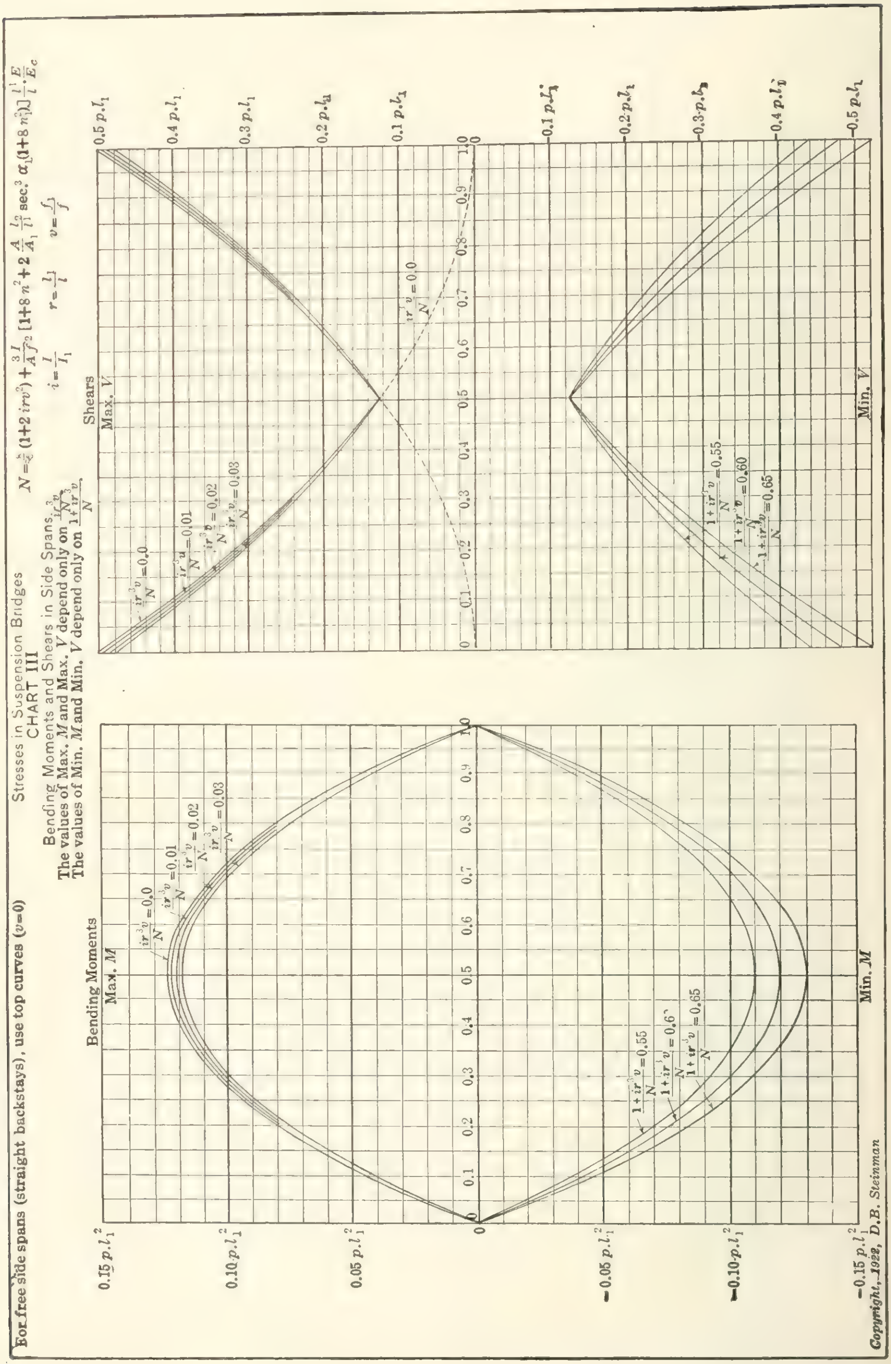


Chart III.—Moments and Shears in Side Spans.

lower curves $\left(\text{for different values of } \frac{1+ir^3v}{N}\right)$ give the minimum shears. (These curves represent the effect of load covering the two other spans.) The values of Total V (for load covering all three spans) may be obtained, if desired, by arithmetically subtracting Min. V from Max. V. The resulting values would be represented by radiating straight lines above the axis.

In this diagram, the plus sign indicates a shear upward on the outer side and downward on the inner side of a section; the minus sign indicates a shear in the opposite direction.

Chart III can also be used for a side span not suspended from the backstays (Type $2F$), or for any independent simple span. The maximum bending moments produced by uniform load are given by the top curve in the left-hand diagram; the minimum bending moments are zero. The maximum shears produced by uniform load are given by the top curve in the right-hand diagram; the minimum shears are given by the dotted continuation curve in the same diagram.

Where locomotive loadings with axle-concentrations are specified, the equivalent uniform loads are to be used for p in these charts.

INDEX

Numbers refer to pages. Illustrations are indicated by an asterisk (*) after page number. For definitions of symbols, consult pages listed under the index word *Notation*.

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